MetStröm, June 2011

Dynamic modeling of multi-scale aspects of turbulent boundary layers

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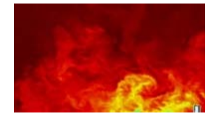
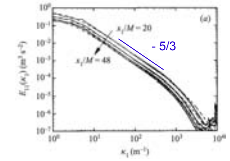
Turbulent flow: multiscale





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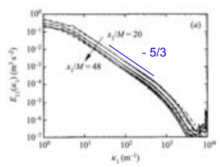
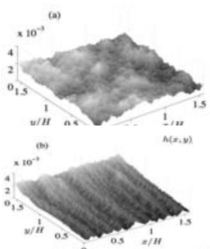
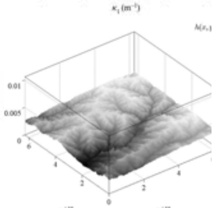
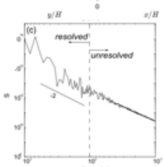


Typical rough surfaces: multiscale





U.S. Geological Survey (USGS): Seamless Server Digital Elev.Map: Llano, TX (30°31'N -99°-100°W)



From: Passalacqua et al (WRR 42, 2006)

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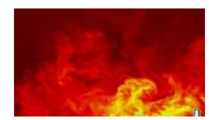

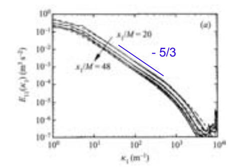

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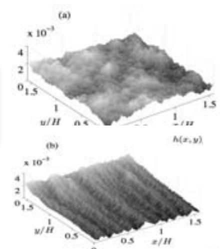

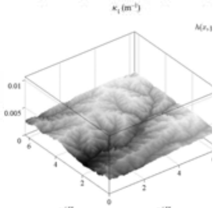

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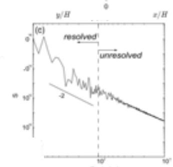

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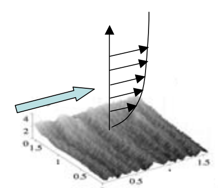
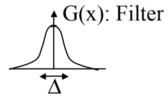
Modeling turbulent flow over rough surfaces

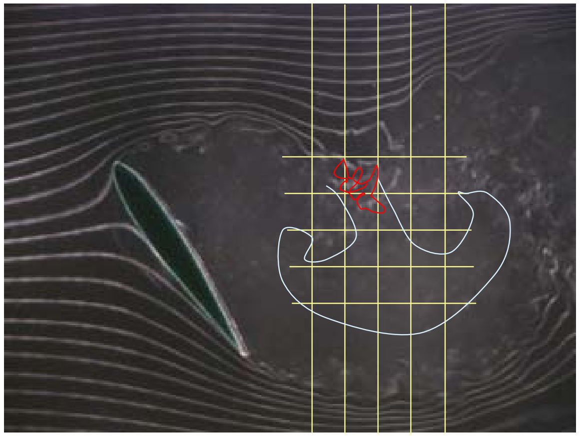


work done with Will Anderson
see JFM article, 2011

Large-eddy-simulation (LES) and filtering:

$$\frac{\partial \tilde{u}_j}{\partial t} + \tilde{u}_k \frac{\partial \tilde{u}_j}{\partial x_k} = -\frac{\partial \tilde{p}}{\partial x_j} + \nu \nabla^2 \tilde{u}_j - \frac{\partial}{\partial x_k} \tau_{jk}$$

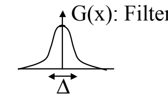


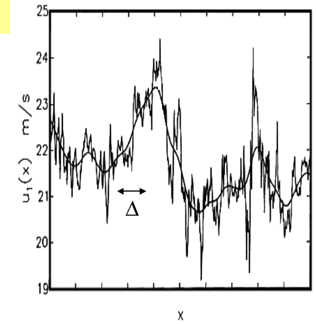


Large-eddy-simulation (LES) and filtering:

N-S equations:

$$\frac{\partial u_j}{\partial t} + \frac{\partial u_k u_j}{\partial x_k} = -\frac{1}{\rho}\frac{\partial p}{\partial x_j} + \nu \nabla^2 u_j \qquad \frac{\partial u_j}{\partial x_j} = 0$$





Filtered N-S equations:

$$\frac{\partial \tilde{u}_j}{\partial t} + \frac{\partial \widetilde{u_k u_j}}{\partial x_k} = -\frac{1}{\rho}\frac{\partial \tilde{p}}{\partial x_j} + \nu \nabla^2 \tilde{u}_j$$

$$\frac{\partial \tilde{u}_j}{\partial t} + \tilde{u}_k \frac{\partial \tilde{u}_j}{\partial x_k} = -\frac{1}{\rho}\frac{\partial \tilde{p}}{\partial x_j} + \nu \nabla^2 \tilde{u}_j - \frac{\partial}{\partial x_k} \tau_{jk}$$

where SGS stress tensor is:

$$\tau_{ij} = \widetilde{u_i u_j} - \tilde{u}_i \tilde{u}_j$$

Most common modeling approach: eddy-viscosity

$$\tau_{ij}^{d} = -\nu_{sgs}\left(\frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i}\right) = -2\nu_{sgs}\tilde{S}_{ij}$$

Functional form in analogy to kinetic theory
of gases (Chapman-Enskog expansions, etc..
"Eddies ~ molecules" (???)

Limitations of the basic eddy-viscosity rationale:

$$\tau_{ij}^{d} = -\nu_{sgs}\left(\frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i}\right) = -2\nu_{sgs}\tilde{S}_{ij}$$

Turbulence is not like a "can of sand"

Limitations of the basic eddy-viscosity rationale:

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Turbulence is not like a "can of sand"

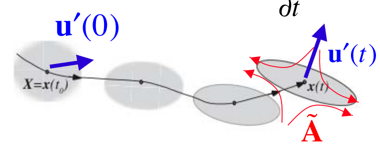
but more like a "can of worms"

Still, in LES eddy-viscosity seems to work "better than it should"
Also, many models need eddy-viscosity additions in ad-hoc "regularizations"
Next 2 slides: an excuse for eddy-viscosity via "fluid dynamics" arguments

A more "fluid-mechanical" rationale for basic eddy-viscosity:

$$\tau_{ij}^d \equiv \left(\widetilde{u_i u_j} - \tilde{u}_i\tilde{u}_j\right)^d \approx \left(\widetilde{u_i' u'}\right)^d$$

$$\frac{\partial(\tilde{u}_i + u_i')}{\partial t} + (\tilde{u}_k + u_k')\frac{\partial(\tilde{u}_i + u_i')}{\partial x_k} = forces$$



$$\frac{du_i'}{dt} = -\frac{\partial \tilde{u}_i}{\partial x_k}u_k' + [forces' - \nabla(u'u')]$$

"Production-only" approximation: $\frac{d\mathbf{u}'}{dt} \approx -\tilde{\mathbf{A}}\bullet\mathbf{u}'$,

(stretching and tilting of vel.
fluctuation by large-scale velocity gradients
- consistent with vortex stretching)

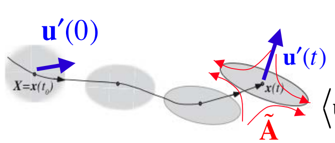
$$\tilde{A}_{ik} = \frac{\partial \tilde{u}_i}{\partial x_k}$$

See Li, Chevillard, Eyink & Meneveau (Phys. Rev. E 2009)

A more "fluid-mechanical" rationale for basic eddy-viscosity:

$$\frac{d\mathbf{u}'}{dt} \approx -\tilde{\mathbf{A}}\bullet\mathbf{u}' \longrightarrow \mathbf{u}'(t) \approx \left(\mathbf{I} - \tilde{\mathbf{A}}t + \frac{1}{2}(\tilde{\mathbf{A}}t)^2 - \ldots\right)\bullet\mathbf{u}'(0)$$

$$\mathbf{u}'\mathbf{u}'^T \approx \left[\left(\mathbf{I} - \tilde{\mathbf{A}}t\right)\bullet\mathbf{u}'(0)\right]\left[\mathbf{u}'(0)\bullet\left(\mathbf{I} - \tilde{\mathbf{A}}^T t\right)\right]$$



$$\left\langle \mathbf{u}'\mathbf{u}'^T \middle| \tilde{\mathbf{A}} \right\rangle \approx \left(\mathbf{I} - \tilde{\mathbf{A}}t\right) \bullet \left\langle \mathbf{u}'\mathbf{u}'^T \middle| \tilde{\mathbf{A}} \right\rangle_{t=0} \bullet \left(\mathbf{I} - \tilde{\mathbf{A}}^T t\right)$$

$$isotropy: \quad \left(c_e\Delta|\tilde{S}|\right)^2 \mathbf{I}$$

$$\left\langle \mathbf{u}'\mathbf{u}'^T \middle| \tilde{\mathbf{A}} \right\rangle_{t_A} \approx \left(c_e\Delta|\tilde{S}|\right)^2 \left((\mathbf{I} - \tilde{\mathbf{A}}t_A)\bullet(\mathbf{I} - \tilde{\mathbf{A}}^T t_A)\right) \approx \left(c_e\Delta|\tilde{S}|\right)^2 \left(\mathbf{I} - \underbrace{(\tilde{\mathbf{A}} + \tilde{\mathbf{A}}^T)}_{2\,\tilde{\mathbf{S}}}\, t_A + O(t^2)\right)$$

$$\tau_{ij}^d = \left\langle \mathbf{u}'\mathbf{u}'^T \middle| \tilde{\mathbf{A}} \right\rangle_{t_A}^d \approx -2\underbrace{\left(c_e\Delta|\tilde{S}|\right)^2 t_A}_{\nu_{sgs}}\ \tilde{\mathbf{S}}$$

A more "fluid-mechanical" rationale for basic eddy-viscosity:

$$\tau_{ij}^d = \left\langle \mathbf{u}'\mathbf{u}'^T \right\rangle_{t_A}^d \approx -2\underbrace{\left(c_e\Delta|\tilde{S}|\right)^2 t_A}_{\nu_{sgs}}\ \tilde{\mathbf{S}}$$

choosing $\quad t_A \propto \frac{1}{|\tilde{S}|}$

$\nu_{sgs} = (c_s\Delta)^2\, |\tilde{S}| \quad$ c_S: **"Smagorinsky coefficient"**

Smagorinsky (1963) = **scale-aware** parameterization

practicality: accept approximations etc. but how to select coefficient?

$$\tau_{ij}^{d} = -2\left(c_s\Delta\right)^2 |\tilde{S}| \tilde{S}_{ij}$$

c_s: "Smagorinsky coefficient"

c_s=0.16 works well for isotropic, high Reynolds number turbulence  Lilly-Deardorff (1960s, 70s)

In practice (complex flows), must adapt to local physics

practicality: accept approximations etc. but how to select coefficient?

$$\tau_{ij}^{d} = -2\left(c_s\Delta\right)^2 |\tilde{S}| \tilde{S}_{ij}$$

c_s: "Smagorinsky coefficient"

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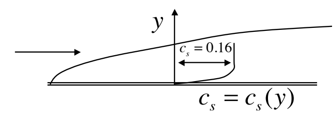
Examples:

1. Transitional pipe flow: had to "manually" change from 0 to 0.16
2. Near wall damping for wall boundary layers (Piomelli et al 1989)



$$c_s = c_s(\mathbf{x}, t)$$

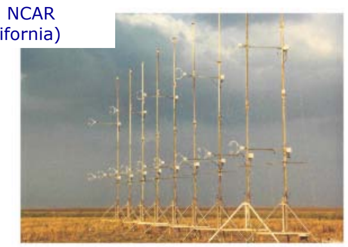
practicality: accept approximations etc. but how to select coefficient?

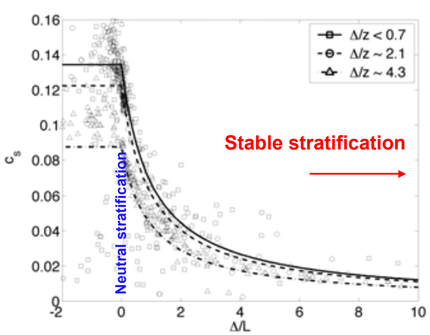
Examples:

3. Effect of atmospheric stability on c_s measured in atmospheric surface layer (Kleissl et al., J. Atmos. Sci. 2003).

$$c_s = \left(\frac{-\langle \tau_{jk} \tilde{S}_{jk} \rangle}{2\Delta^2 \langle |\tilde{S}| \tilde{S}_{ij} \tilde{S}_{ij} \rangle} \right)^{1/2}$$

HATS - 2000 with NCAR (California)

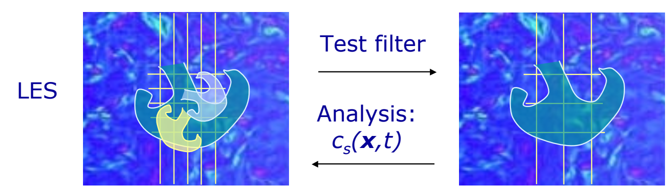




$$c_s = c_s(\mathbf{x}, t)$$

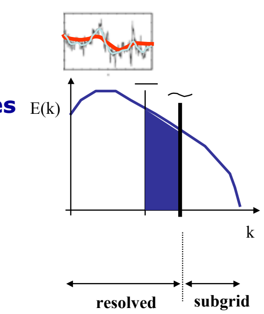
How can this be made less ad-hoc?

Dynamic model (Germano et al., Phys. Fluids 1991)



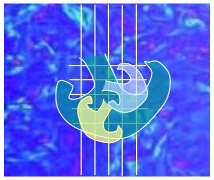
Germano identity:

$\widetilde{\overline{u_i u_j}} \quad = \widetilde{\overline{u_i u_j}}$ **Equality of total fluxes**



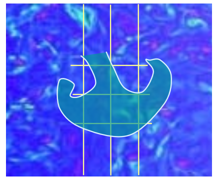
Dynamic model (Germano et al., Phys. Fluids 1991)

LES



Test filter

Analysis: $c_s(\boldsymbol{x},t)$

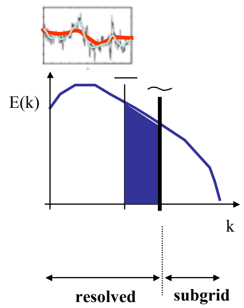


Germano identity:

$$\widetilde{\overline{u_i u_j}} \qquad = \overline{\widetilde{u_i u_j}}$$

Equality of total fluxes

$$\widetilde{\overline{u_i u_j}} - \tilde{\bar{u}}_i \tilde{\bar{u}}_j = \overline{\widetilde{u_i u_j}} \qquad - \bar{\tilde{u}}_i \bar{\tilde{u}}_j$$



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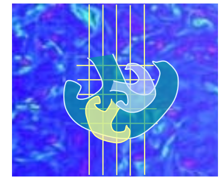

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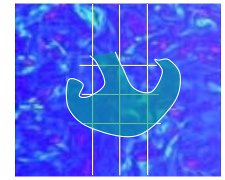

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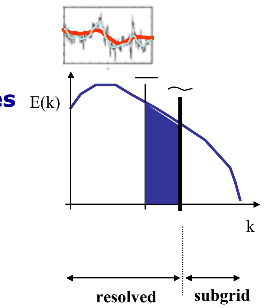
$$\underbrace{\widetilde{\overline{u_i u_j}} - \tilde{\bar{u}}_i \tilde{\bar{u}}_j}_{T_{ij}} = \underbrace{\overline{\widetilde{u_i u_j}} - \overline{\tilde{u}_i}\,\overline{\tilde{u}_j}}_{\bar{\tau}_{ij}} + \underbrace{\overline{\tilde{u}_i \tilde{u}_j} - \bar{\tilde{u}}_i \bar{\tilde{u}}_j}_{L_{ij}}$$

$$T_{ij} = \bar{\tau}_{ij} + L_{ij}$$



Dynamic model (Germano et al., Phys. Fluids 1991)

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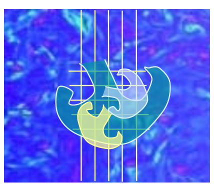

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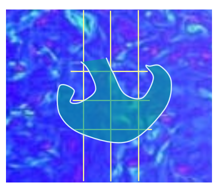

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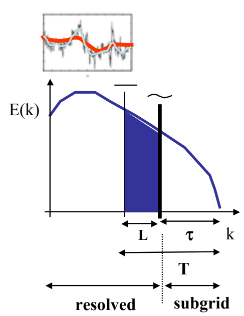
Equality of total fluxes

$$T_{ij} = \bar{\tau}_{ij} + L_{ij}$$

Refined versions:

Lagrangian dynamic model, scale-dependent

- M, Lund & Cabot (JFM 1996)
- Anderson & M (FTC 1999)
- Porté-Agel et al. (JFM 2000)
- Bou-Zeid et al. (Phys Fluids 2005)



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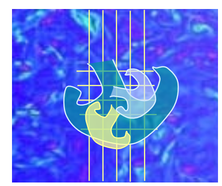

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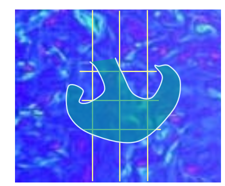

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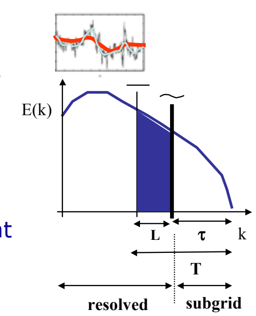
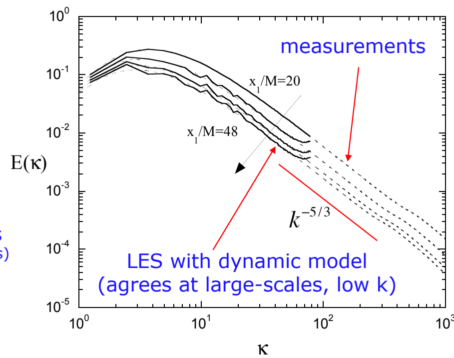
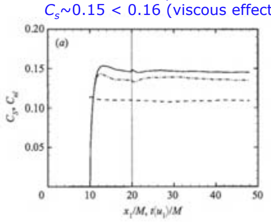


Applications of dynamic subgrid-scale model

Decaying isotropic turbulence (compare with measurements in wind tunnel)
(Kang, Chester & M, JFM 2003)

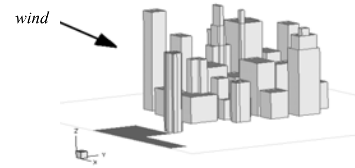


Coefficient computed from LES
C_s~0.15 < 0.16 (viscous effects)

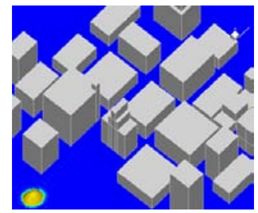


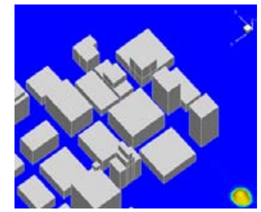
Applications of dynamic subgrid-scale model

Downtown Baltimore:



Momentum and scalar transport equations solved using LES and Lagrangian dynamic subgrid model. Buildings are simulated using immersed boundary method.



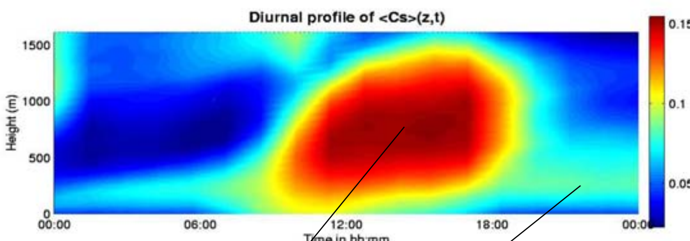


Yu-Heng Tseng, C. Meneveau & M. Parlange, 2006 (Env. Sci & Tech. **40**, 2653-2662)

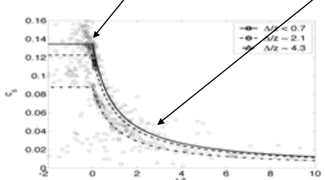
Applications of dynamic subgrid-scale model

LES of diurnal cycle: start stably stratified, then heating….

Resulting dynamic coefficient (averaged):



Consistent with HATS field measurements:

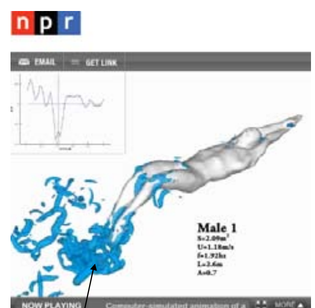


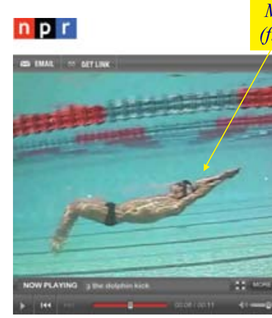
See comparison with CASES-99 (Kumar, Svensson, Holstag, Meneveau & Parlange, J. Applied Met & Climatology **49**, 1496; 2010)

Applications of dynamic subgrid-scale model

Human swimming: vortex flow structures

(Rajat Mittal, 2008 - consulting for US Olympic team)



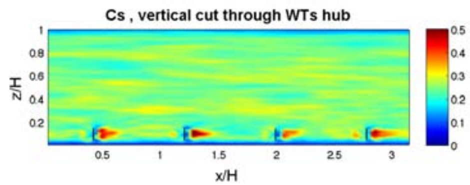


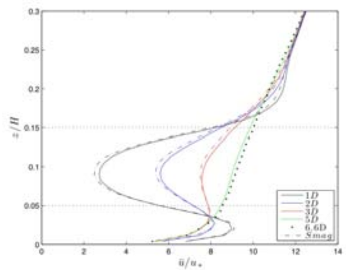
Michael Phelps (from Baltimore)

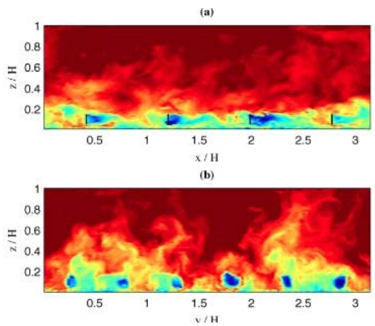
Iso-vorticity surfaces
thrust-force-circulation

Applications of dynamic subgrid-scale model

Large Eddy Simulation of wind-turbine arrays (actuator disk)



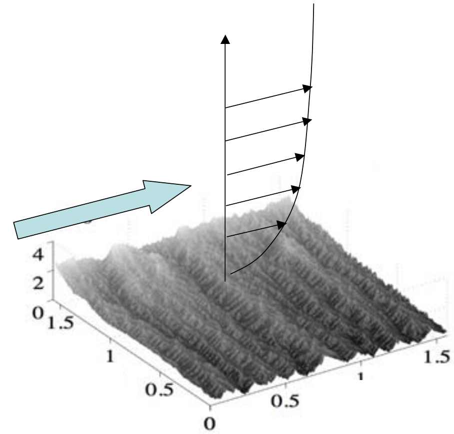




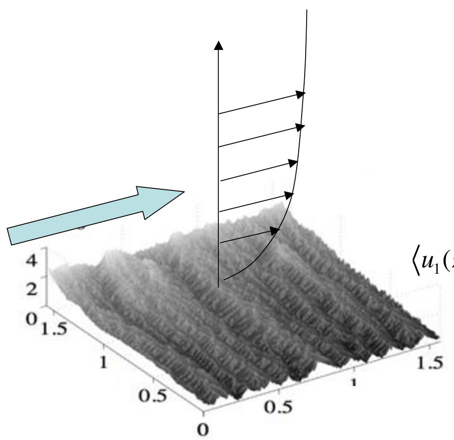
streamwise velocity contours

Calaf, C.M. & Meyers, Phys. Fluids, 2010

LES modeling of flow over multi-scale rough surfaces:

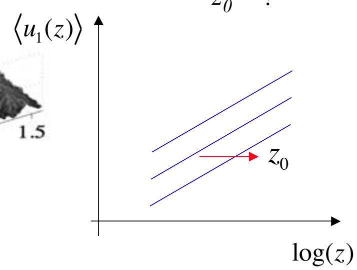


LES modeling of flow over multi-scale rough surfaces:

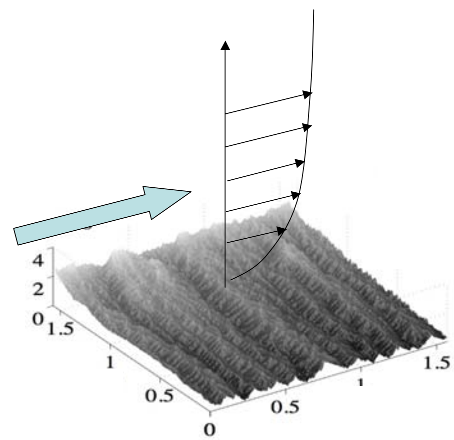


$$\frac{\langle u_1(z)\rangle}{u_*} = \frac{1}{\kappa}\log\left(\frac{z}{z_0}\right)$$

$z_0 = ?$



LES modeling of flow over multi-scale rough surfaces:



$$\frac{\langle u_1(z)\rangle}{u_*} = \frac{1}{\kappa}\log\left(\frac{z}{z_0}\right)$$

$z_0 = ?$

$$z_0^{\Delta} = \alpha\ \sigma_h^{\Delta}$$

"SGS hydrodynamic roughnes scale is proportional to the **SGS rms** of the height *h′(x,y)* variation"

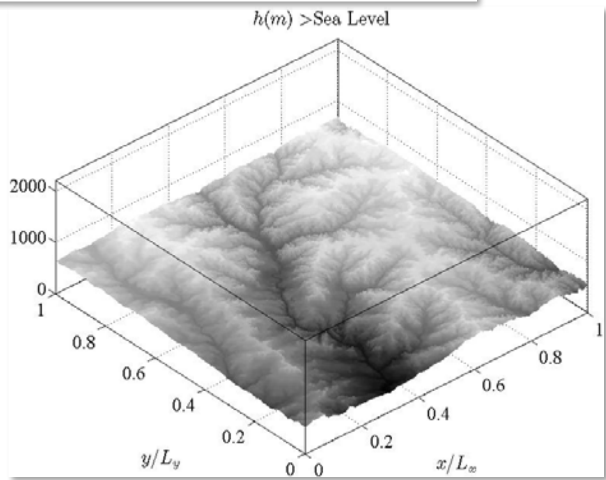
practicality: accept approximations etc. but how to select coefficient α ???

α: hydrodynamic roughness parameter

Geomorphological evolved fluvial landscape:

$$\frac{\langle \tilde{u}_1(z) \rangle}{u_*} = \frac{1}{\kappa} log \left[\frac{z}{z_0} \right]$$

$\alpha = ?$



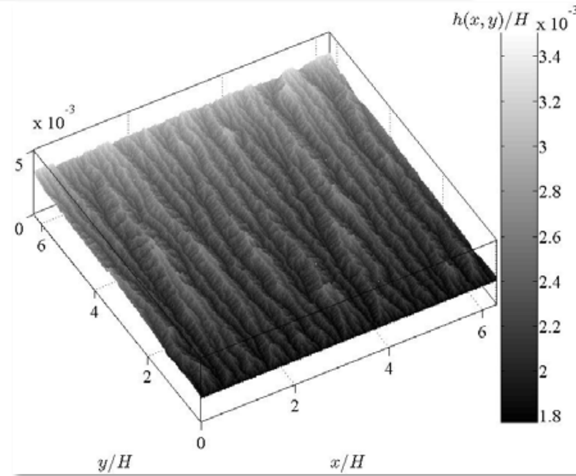
U.S. Geological Survey (USGS): **Seamless Server**
Digital Elevation Map: Llano, TX (30°:31°N -99°:-100°W)

Numerically eroded (KPZ) surface:

$$\frac{\langle \tilde{u}_1(z) \rangle}{u_*} = \frac{1}{\kappa} log \left[\frac{z}{z_0} \right]$$

$z_0 = ?$

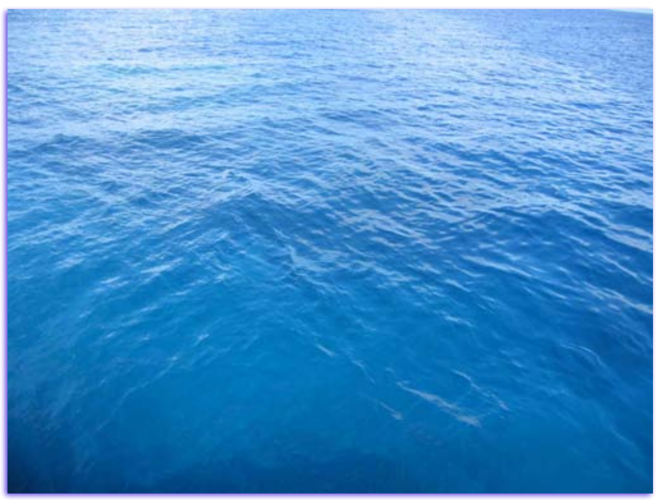
$\alpha = ?$



Numerical solution of KPZ equation for fluvial landscape evolution
(data courtesy of P. Passalacqua and F. Porté-Agel)

Ocean surface:

$z_0 = ?$

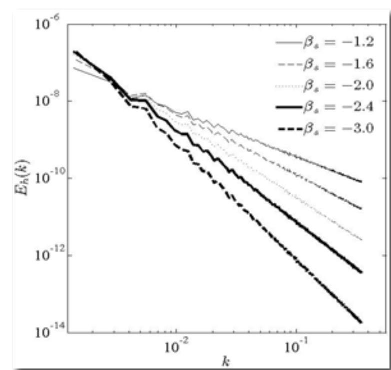


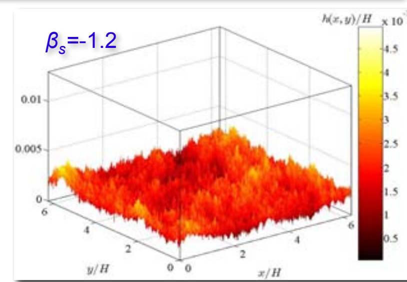
Developed ocean surface spectral slope, $\beta_s \approx -3$

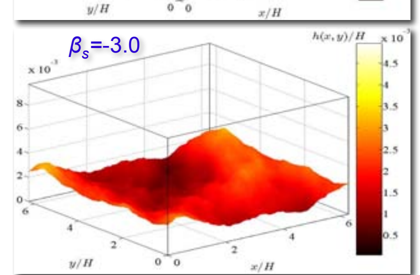
LES of atmospheric boundary layer flow over surfaces created using random-phase Fourier modes and power-law spectra:

$$h(x,y) = \sum_{\underline{k}} c k^{-\beta_s/2} e^{i(\underline{k} \cdot \underline{x} + \varphi)}$$

Radial Spectra of h(x,y)







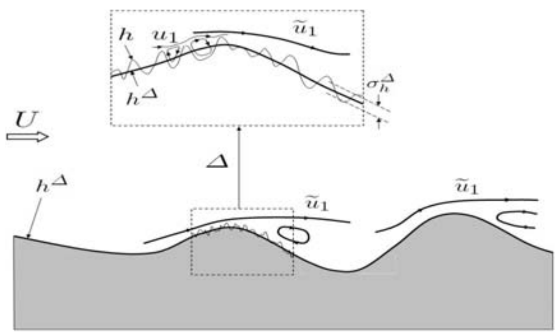
Dynamic surface roughness model:

Anderson & CM,
Journal of Fluid Mechanics, 2011

Resolved pressure field $\tilde{p}^{w}(x,y)$

LES of ABL: law-of-the-wall boundary conditions $\tau_{i3}^{w,\Delta} = -\left[\frac{\kappa U^{\Delta}}{ln\left(z/z_0^{\Delta}\right)}\right]^2 \frac{\tilde{u}_i}{U^{\Delta}},\ i=1,2$

Total surface force: $F_i = -\iint\limits_S \tilde{p}^{w}\tilde{n}_i dS + \iint\limits_S \tau_{ij}^{w,\Delta}\tilde{n}_j dS$



Related ideas:
Nakayama, Hori &
Street (CTR 2004)

Dynamic surface roughness model:

Same total force at Δ or 2 Δ (**fundamental equation**):

$$F_i = -\iint\limits_S \tilde{p}^{w}\tilde{n}_i dS + \iint\limits_S \tau_{ij}^{w,\Delta}\tilde{n}_j dS = -\iint\limits_S \hat{\tilde{p}}^{w}\hat{\tilde{n}}_i dS + \iint\limits_S \tau_{ij}^{w,2\Delta}\hat{\tilde{n}}_j dS$$

Resolved pressure field $\tilde{p}^{w}(x,y)$

Resolved test-filtered pressure field $\hat{\tilde{p}}^{w}(x,y)$

Wall stress at Δ: $\tau_{i3}^{w,\Delta} = -\left[\frac{\kappa U^{\Delta}}{ln\left(z/z_0^{\Delta}\right)}\right]^2 \frac{\tilde{u}_i}{U^{\Delta}},\ i=1,2$

Wall stress expressed at 2 Δ: $\tau_{i3}^{w,2\Delta} = -\left[\frac{\kappa U^{2\Delta}}{ln\left(z/z_0^{2\Delta}\right)}\right]^2 \frac{\hat{\tilde{u}}_i}{U^{2\Delta}},\ i=1,2$

Dynamic surface roughness model:

$$\left\langle \tau_{13}^{\Delta}\big|_{wall} \right\rangle + \left\langle \int_S \tilde{p}\tilde{n}_1 dS \right\rangle \frac{1}{S} = \left\langle \tau_{13}^{2\Delta}\big|_{wall} \right\rangle + \left\langle \int_S \hat{\tilde{p}}\hat{\tilde{n}}_1 dS \right\rangle \frac{1}{S}$$

$$\left\langle \left[\frac{\kappa U^{\Delta}}{ln\left(\frac{\Delta_z/2 - h^{\Delta}}{\alpha\sigma_h^{\Delta}}\right)}\right]^2 \frac{\tilde{u}_1}{U^{\Delta}} \right\rangle + \left\langle \tilde{p}^{w}\frac{\partial \tilde{h}}{\partial x_1} \right\rangle = \left\langle \left[\frac{\kappa U^{2\Delta}}{ln\left(\frac{\Delta_z/2 - h^{2\Delta}}{\alpha\sigma_h^{2\Delta}}\right)}\right]^2 \frac{\hat{\tilde{u}}_1}{U^{2\Delta}} \right\rangle + \left\langle \hat{\tilde{p}}^{w}\frac{\partial \hat{\tilde{h}}}{\partial x_1} \right\rangle$$

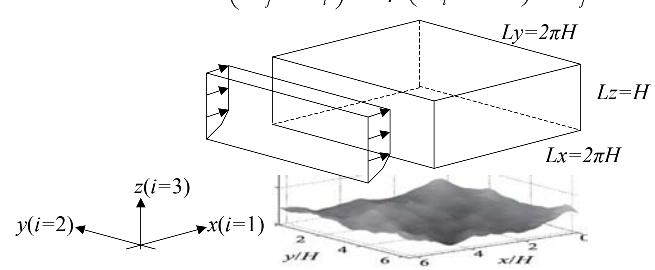
In LES: only unknown is α
(solve e.g. using bi-section method)

Large Eddy Simulation of boundary layers over rough surfaces:

Implement into ABL LES code (neutral)

- Pseudospectral in horizontal, 2nd-order finite difference in vertical (Moeng 1984-type, Albertson and Parlange, 1999: *Water Resour. Res.*)

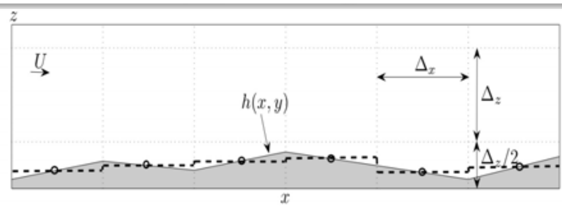
$$\frac{\partial \tilde{u}_i}{\partial t} + \tilde{u}_j\left(\frac{\partial \tilde{u}_i}{\partial x_j} - \frac{\partial \tilde{u}_j}{\partial x_i}\right) = -\frac{1}{\rho}\left(\frac{\partial \tilde{p}}{\partial x_i} + \delta_{i1}\Pi\right) - \frac{\partial \tau_{ij}}{\partial x_j} + f_i$$



SGS closure: Lagrangian scale-dep. dynamic model (E. Bou-Zeid et al., 2005: *Phys. Fluids*)

For resolved drag force, instead of terrain-following mesh approach, where p(x,y) is well resolved, here we use a newly proposed **surface-gradient drag model** for simpler non-terrain-following code…
for applications of horizontally resolved but vertically unresolved surfaces

(Anderson and Meneveau: *Boundary Layer-Meteorology 2010*)



Extract incoming momentum flux of block frontal area

$$\boldsymbol{F}^{\Delta}_{res} = \rho\int_A \tilde{p}^w \tilde{\boldsymbol{n}} dS \approx \rho\int_A \tilde{\boldsymbol{u}}\left(\tilde{\boldsymbol{u}}\cdot\tilde{\boldsymbol{n}} dS\right) \approx -\rho\tilde{\boldsymbol{u}}\left(\tilde{\boldsymbol{u}}\cdot\nabla\tilde{h}\Delta_x\Delta_y\right) = -\rho\Delta_x\Delta_y\tilde{\boldsymbol{u}}\left(\tilde{\boldsymbol{u}}\cdot\nabla\tilde{h}\right)$$

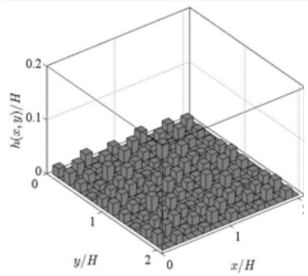
$$\boldsymbol{f}^{\Delta} = \frac{1}{\rho}\frac{1}{\Delta V}\boldsymbol{F}^{\Delta}_{res} = -\frac{1}{\Delta_z}\tilde{\boldsymbol{u}}\; R\left(\tilde{\boldsymbol{u}}\cdot\nabla\tilde{h}\right)$$

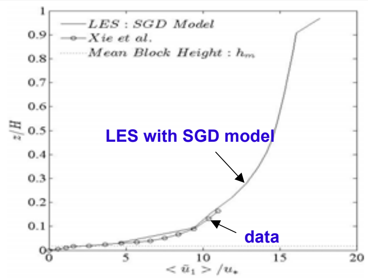
Ramp-function: for front facing only

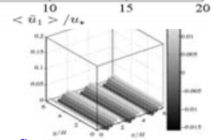
$$f_i^{\Delta}\left(x,y,\Delta_z/2\right) = -\frac{1}{\Delta_z}\tilde{u}_i R\left(\tilde{u}_k\frac{\partial\tilde{h}}{\partial x_k}\right), i=1,2$$

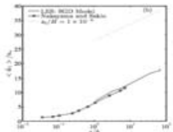
$$f_3^{\Delta} = 0$$

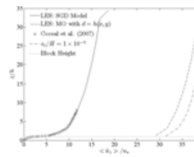
Validation of SGD model on "large-scale" single-length roughnesses:
Flow over array of staggered spatially varying height blocks (Xie et al., 2008)









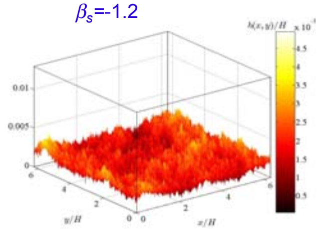


Other comparisons made with literature datasets for flow over
- sinusoidal roughness elements (Nakayama and Sakio, 2002)
- "egg carton" roughness (3-d. ellipsoids) (Bhaganagar et al., 2004)
- staggered array of uniform height blocks (Coceal et al., 2007)
- non-staggered array of uniform height blocks (Kanda et al., 2004)

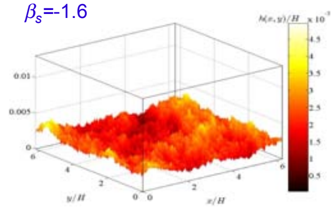
→ **Good Agreement**

Suite of LES cases with bottom surface with different spectral exponents:

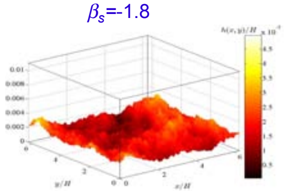
β_s=-1.2



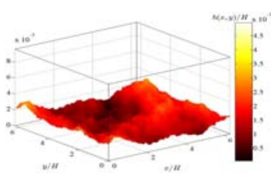
β_s=-1.6



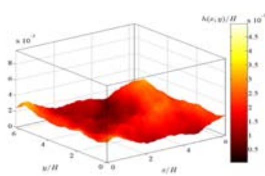
β_s=-1.8



β_s=-2.4



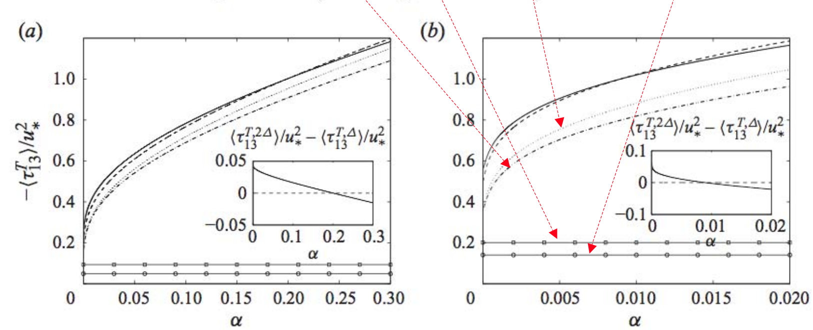
β_s=-3.0



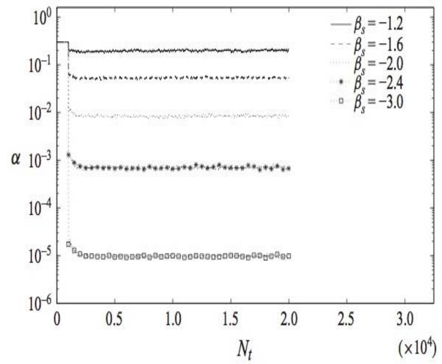
Bisection Method Solution: DSR Model (32³ LES)

$$\left\langle\left[\frac{\kappa U^{\Delta}}{ln\left(\frac{\Delta_z/2-h^{\Delta}}{\alpha\sigma_h^{\Delta}}\right)}\right]^2\frac{\tilde{u}_1}{U^{\Delta}}\right\rangle + \left\langle\tilde{u}_1 R\left(\tilde{u}_k\frac{\partial\tilde{h}}{\partial x_k}\right)\right\rangle = \left\langle\left[\frac{\kappa U^{2\Delta}}{ln\left(\frac{\Delta_z/2-h^{2\Delta}}{\alpha\sigma_h^{2\Delta}}\right)}\right]^2\frac{\hat{\tilde{u}}_1}{U^{2\Delta}}\right\rangle + \left\langle\hat{\tilde{u}}_1 R\left(\hat{\tilde{u}}_k\frac{\partial\hat{\tilde{h}}}{\partial x_k}\right)\right\rangle$$

Dynamic surface roughness model for LES



Results: time-evolution of a and dependence on surface spectral slope

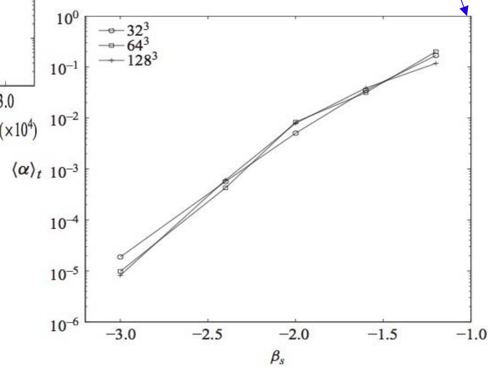


For $\beta_S \geq -1$, the total SGS variance diverges

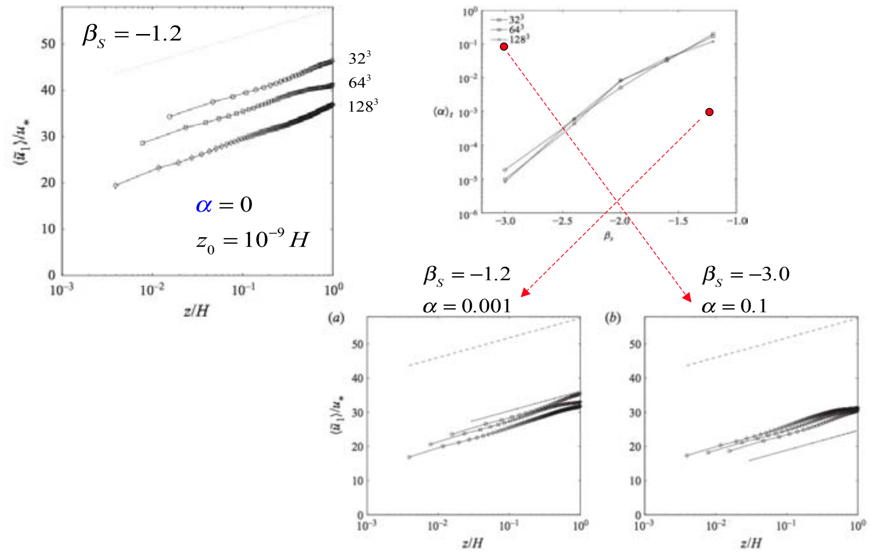
$$z_0^{\Delta} = \alpha \sigma_h^{\Delta}$$

$$E_h(k) \sim k^{-\beta_s}$$

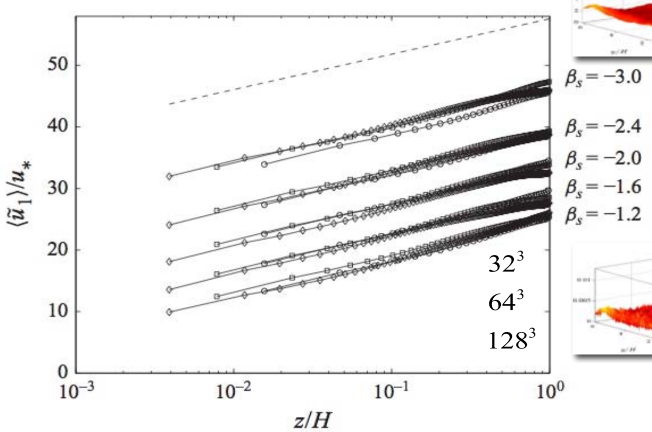
$$\left(\sigma_h^{\Delta}\right)^2 = \int_{\pi/\Delta}^{\infty} E_h(k)\,dk$$

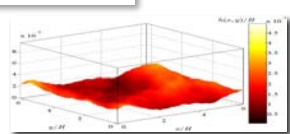


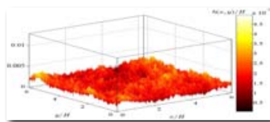
Mean velocity profiles:
resolution-dependence if α=0 or α =“wrong values”



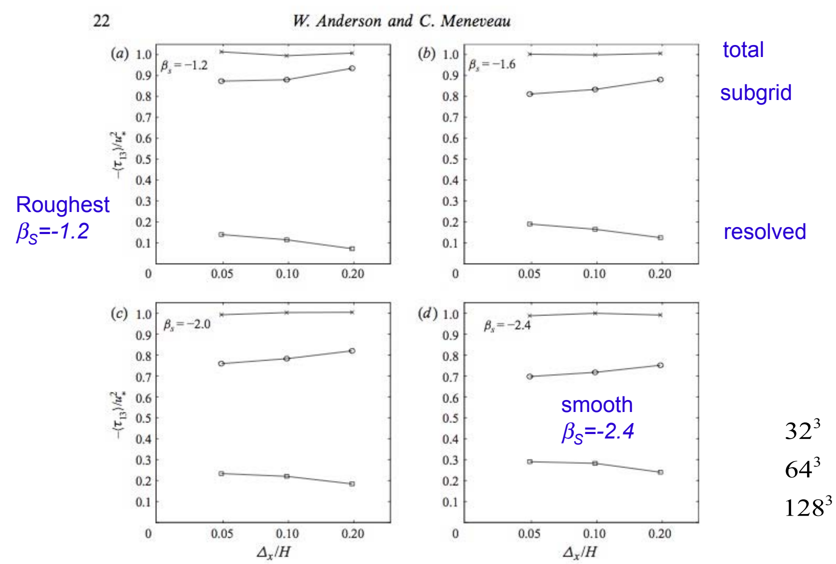
Mean velocity profiles:
Nearly resolution-independent if α =“dynamically determined”



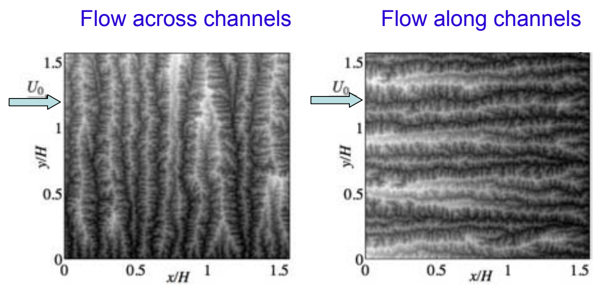




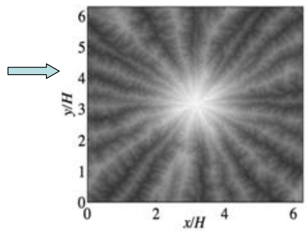
Balance of total wall stress: resolved and subgrid:



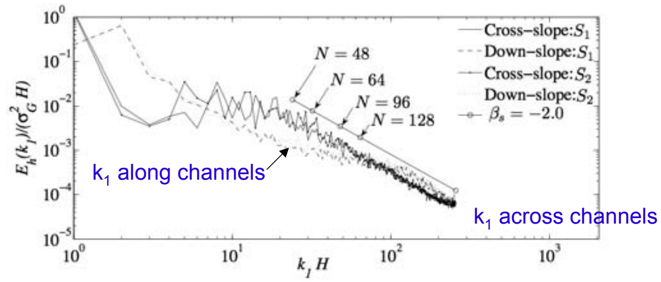
Anisotropic surfaces: Applications to fluvial evolved landscapes (KPZ):



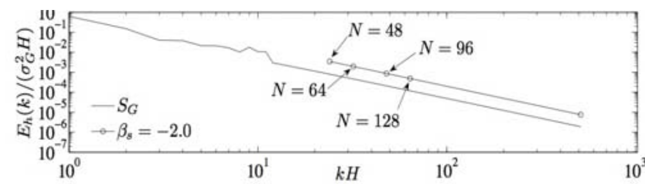
Flow over a Gaussian bump+evolved fluvial landscape (+spectral rescaling k^{-2})



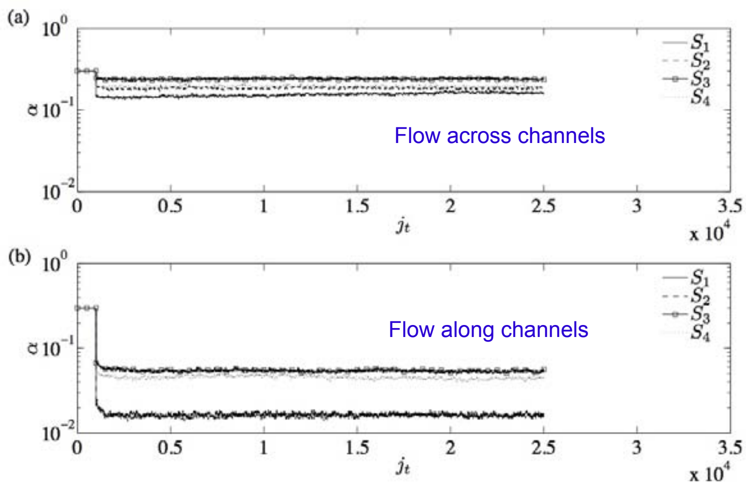
Streamwise spectra of fluvial evolved landscapes (KPZ):



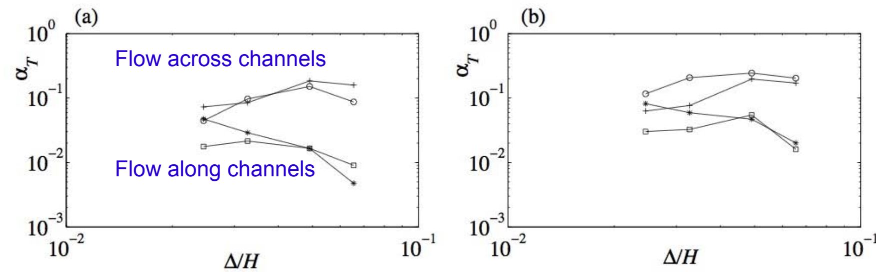
Gaussian bump+evolved fluvial landscape (radial spectrum)

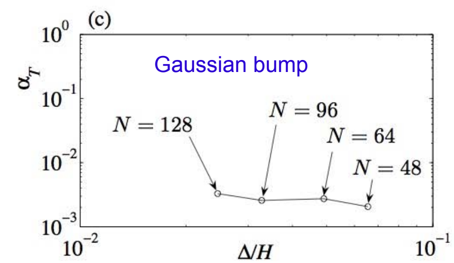


Results: evolution of α

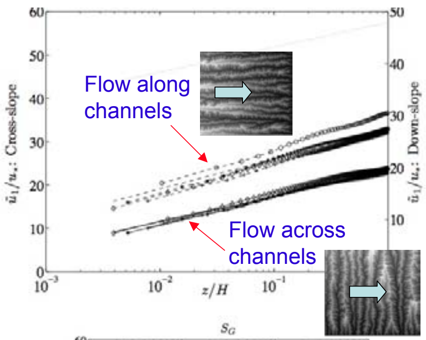


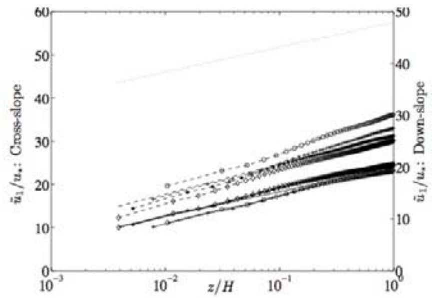
Results: evolution of α

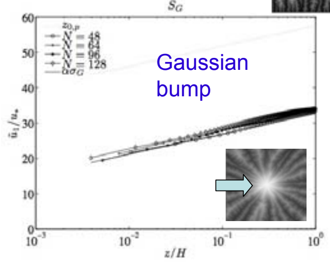




Results: Mean velocity profiles







Conclusion:

Good results for cases when spectrum of surface normal to the flow is power-law (scale-invariance)

Closing:

$$F_i = -\iint_S \tilde{p}^w \tilde{n}_i dS + \iint_S \tau_{ij}^{w,\Delta} \tilde{n}_j dS = -\iint_S \hat{\tilde{p}}^w \hat{\tilde{n}}_i dS + \iint_S \tau_{ij}^{w,2\Delta} \hat{\tilde{n}}_j dS$$

Scale-aware SGS parameterizations:

Must obey generalized Germano identities:

$$\widehat{\widetilde{NL(\boldsymbol{q})}} = NL(\hat{\tilde{\boldsymbol{q}}}) + Mod(\hat{\tilde{\boldsymbol{q}}},\Delta) = \overline{NL(\tilde{\boldsymbol{q}}) + Mod(\tilde{\boldsymbol{q}}, r\Delta)}$$

Constrains parameter values
(variants: multi-scale, averaging domain, stochasticity…)

Thanks

Questions?