International MetStröm-Conference 2011
09.06.2011, Berlin

Application of Quadrature Method of Moments for Sedimentation and Coagulation of Raindrops

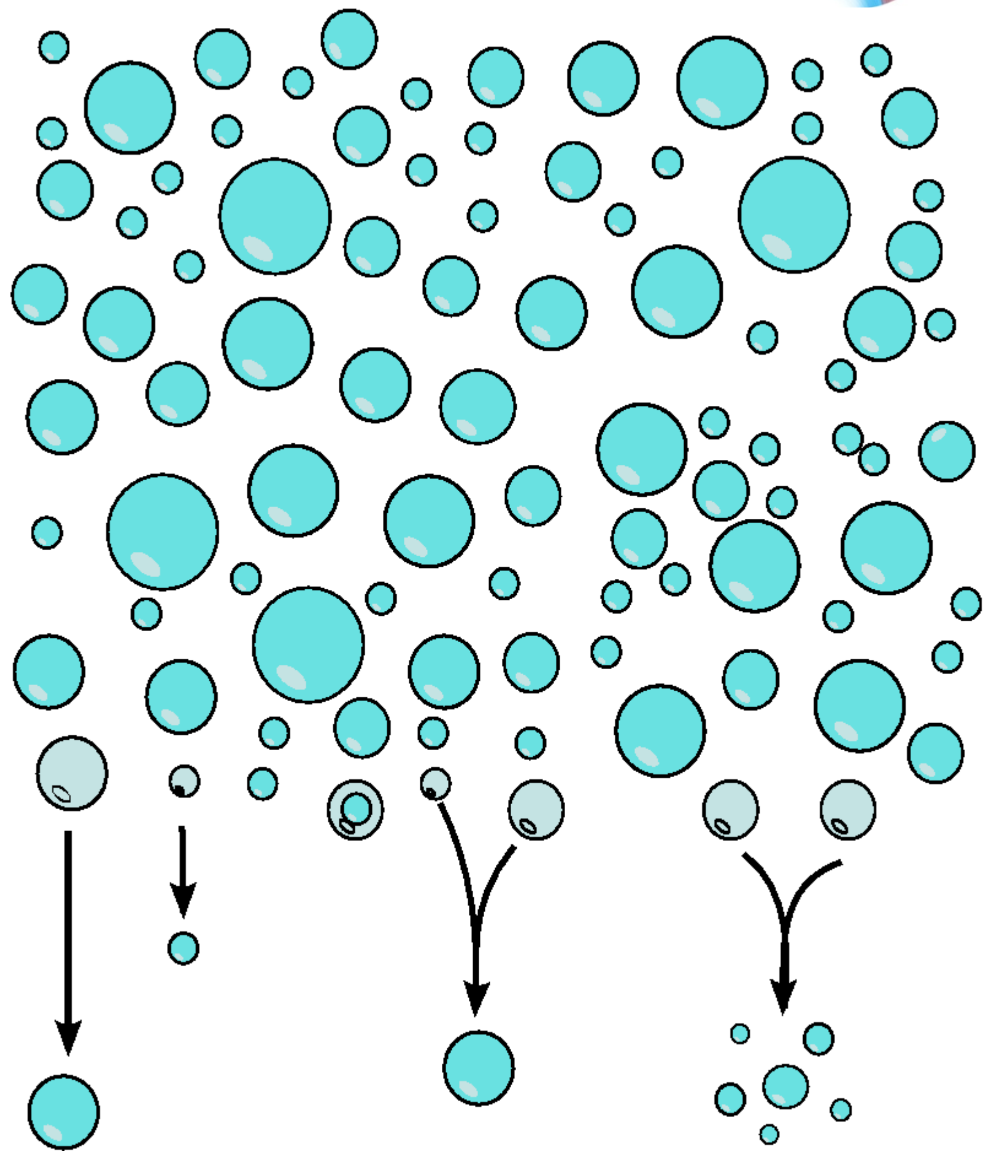
Gary Jasor [1], Ulrike Wacker [2], Klaus Beheng [3] and Wolfgang Polifke [1]

[1] TU München [2] AWI Bremerhaven [3] KIT Karlsruhe

With financial Support of Priority Programm SPP1276 Metström (DFG)
Project: *Multi-scale Modelling of the Population Dynamics of Hydrometeors with Methods of Moments (MoM's)*

Multi-scale Modelling…

- Cloud physics :
  - – sedimentation
  - – coagulation
  - – break-up
  - – evaporation/condensation



Spectral kinetic equation

flow — turbulent subgrid-scale — evaporation

$$\frac{\partial f(r)}{\partial t} + \nabla_x.[f(r)\vec{v}] + \nabla_x.\left[f(r)\overrightarrow{v'}(r)\right] + \frac{\partial[f(r)\dot{r}]}{\partial r}$$

$$=$$

$$\frac{1}{2}\int_{r_i=0}^{r} K(r_i,R)f(r_i)f(R)dr_i \; - f(r)\int_{r_i=0}^{\infty} K(r,r_i)f(r_i)dr_i$$

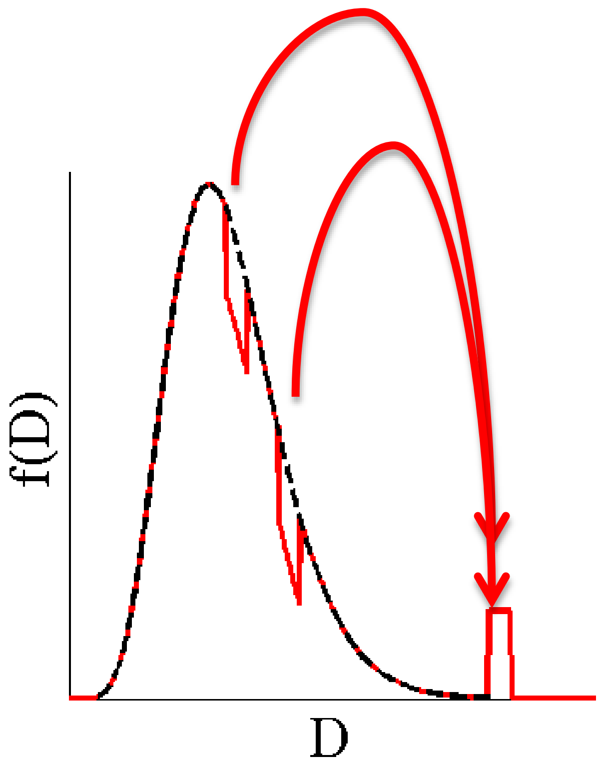
formation — “destruction“

$K(R_1,R_2)$: efficiency with which two particles with diameter R_1 resp. R_2 will merge to form a bigger drop

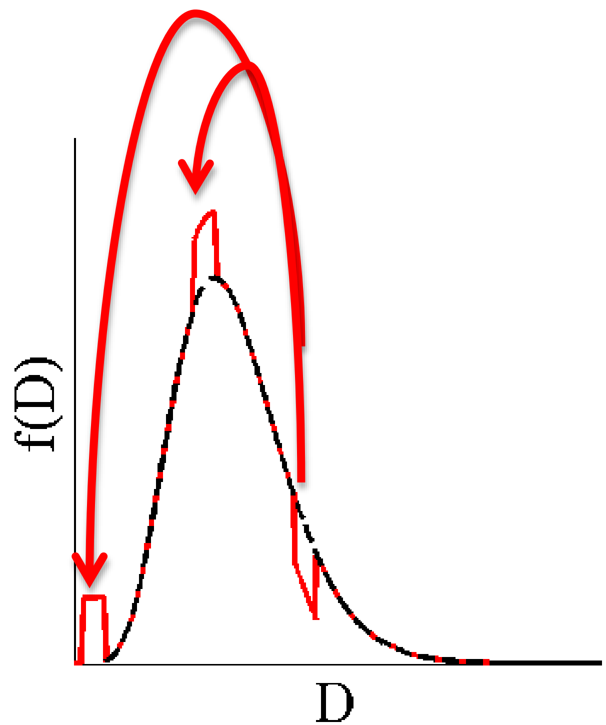
$f(r,\vec{x},t)$: particle size density function in $m^{-1}m^{-3} = m^{-4}$

Population Dynamics of Hydrometeors

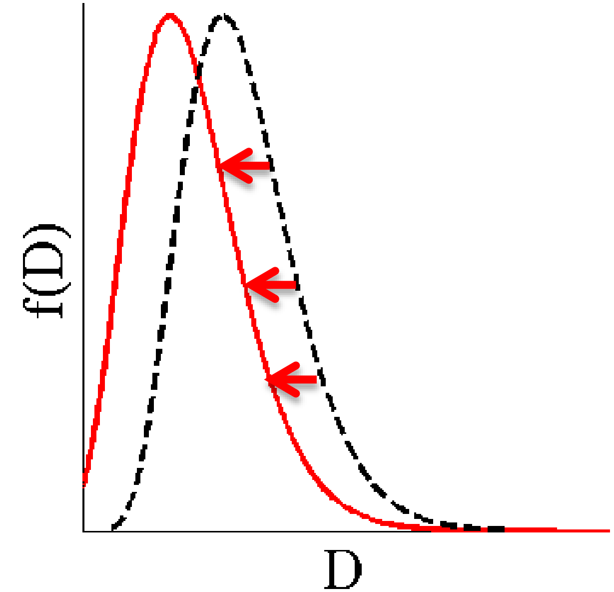
Coagulation



Break-up



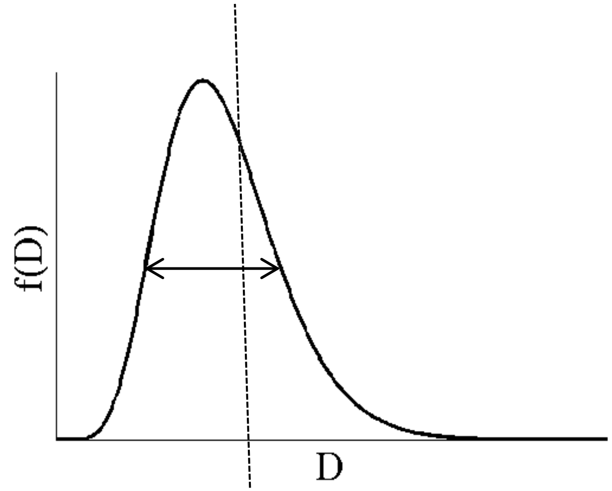
Evaporation



Methods of Moments (MoM's)

- *f(D)* : particle size distribution (PSD)

- $M^{(k)} = \int D^k f(D) dD$
  Moment of k-th order.



Particle Size Distribution

- Physical Meaning of the moments :
  - $M^{(0)} = N$ — number of particles
  - $\pi M^{(2)} = \pi \int D^2 f(D) dD$ — total surface area of the particles
  - $\frac{\pi}{6} M^{(3)} = \frac{\pi}{6} \int D^3 f(D) dD$ — total volume of the particles (volume fraction)
  - $L = \frac{\pi}{6} \rho_w M^{(3)}$ — water content
  - $M^{(6)} \propto Z$ — proportional to the reflectivity (albedo)

Outline

Introduction

Quadrature Method of Moments

- Background
- Gaussian quadrature

Application to 1D cases

- Pure sedimentation
- Sedimentation with coagulation

Spectral Method

- Principle :
  - – discretise the diameter range into n_D classes $[D_i, D_{i+1}[$
  - – solve for each class
  - – sum over these classes to compute the moments

$$M^{(k)} \approx \frac{1}{n_D} \sum_i {D_i}^k$$

- Straightforward but very costly
- Serves as a reference solution when affordable

Spectral kinetic equation

flow | turbulent subgrid-scale | evaporation

$$\frac{\partial f(r)}{\partial t} + \nabla_x . [f(r)\vec{v}] + \nabla_x . \left[f(r)\overrightarrow{v'}(r)\right] + \frac{\partial[f(r)\dot{r}]}{\partial r}$$

$$=$$

$$\frac{1}{2}\int_{r_i=0}^{r} K(r_i,R)f(r_i)f(R)dr_i - f(r)\int_{r_i=0}^{\infty} K(r,r_i)f(r_i)dr_i$$

formation | “destruction“

Simplification for 1D case (over altitude) with diameter as internal coordinate

Spectral kinetic equation

flow

$$\frac{\partial f(r)}{\partial t} + \nabla_x . [f(r)\vec{v}] + \qquad + \qquad = \frac{1}{2}\int_{r_i=0}^{r} K(r_i, R) f(r_i) f(R) dr_i - f(r)\int_{r_i=0}^{\infty} K(r, r_i) f(r_i) dr_i$$

formation

“destruction“

$$\frac{\partial f(D,z,t)}{\partial t} - \frac{\partial V(D,z,t) f(D,z,t)}{\partial z} = S_p$$

Moment transport equation

- Multiply by D^k and integrate over the diameter range

$$\frac{\partial M^{(k)}(z,t)}{\partial t} - \frac{\partial U^{(k)}(z,t)M^{(k)}(z,t)}{\partial z} = S_c$$

with

$$U^{(k)} = \frac{1}{M^{(k)}(z,t)} \int_{D=0}^{+\infty} V(D,z,t)D^k f(D,z,t)dD$$

Actually, $V(D) = \alpha \left(\frac{D}{D_v}\right)^{\beta}$: terminal fall velocity

Closure Problem and Source Terms

- For the moment transport equation : $$\frac{\partial M^{(k)}}{\partial t} - C_v \frac{\partial M^{(k+\beta)}}{\partial z} = S_c$$
  - Knowing $M^{(k)}$ does not imply we know $M^{(k+\beta)}$ (higher order!!)
  - Extra modelling can be required

- For the source term :

$$S_c = \frac{1}{2}\int_{D=0}^{\infty}\int_{D'=0}^{\infty}\left[\left(\left(D^3 - D'^3\right)^{1/3}\right)^k - D^k - D'^k\right] K(D,D') f(D) f(D') dD'\, dD$$

with

$$K(D,D') = \frac{\pi}{4}(D+D')^2 |V(D) - V(D')| E_{eff}(D,D')$$

  - No straightforward term involving a moment

Quadrature MoM (QMoM)

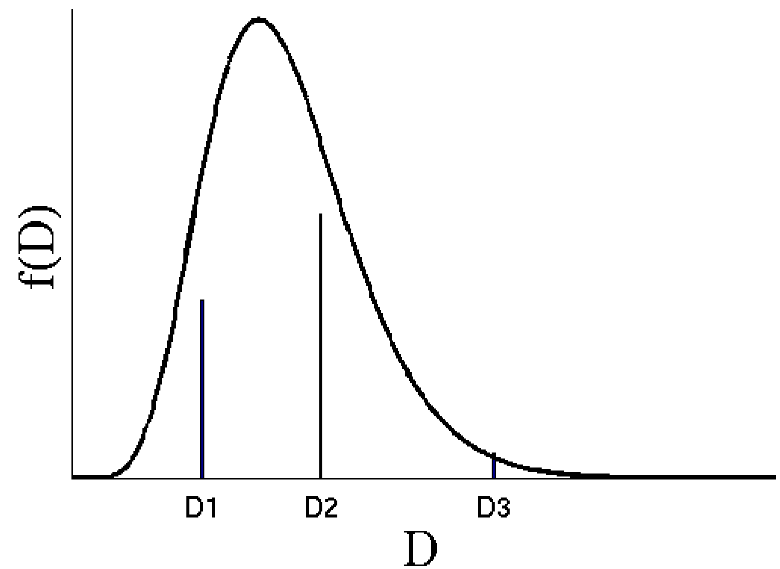
- Principle :
  - Based on Gauss quadrature :
    $\exists \ (D_i, \omega_i)_i \in \mathbb{R}^{2N_q}$,

$$I = \int_0^{+\infty} g(D) f(D) dD \ \approx \sum_{i=1}^{N_q} \omega_i g(D_i)$$

  - Application to $g = D^k$

$$M^{(k)} \approx \sum_{i=1}^{N_q} {D_i}^k \omega_i$$

  - For $N_q = 3$,
    6 prognostic moments needed



Outline

Introduction

Quadrature Method of Moments

- Background
- Gaussian quadrature

Application to 1D cases

- Pure sedimentation
- Sedimentation with coagulation

Study Case

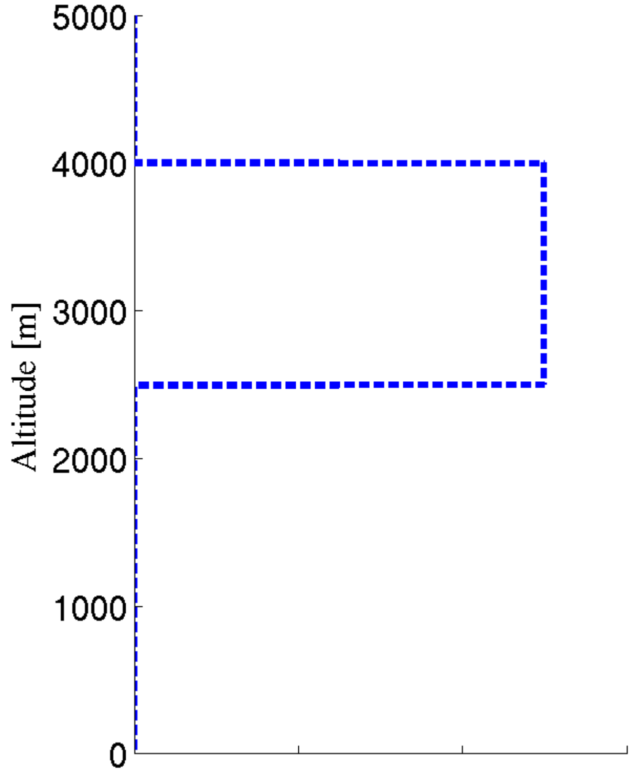
- 1D Moment transport equation

$$\frac{\partial M^{(k)}}{\partial t} - C_v \frac{\partial M^{(k+\beta)}}{\partial z} = 0$$

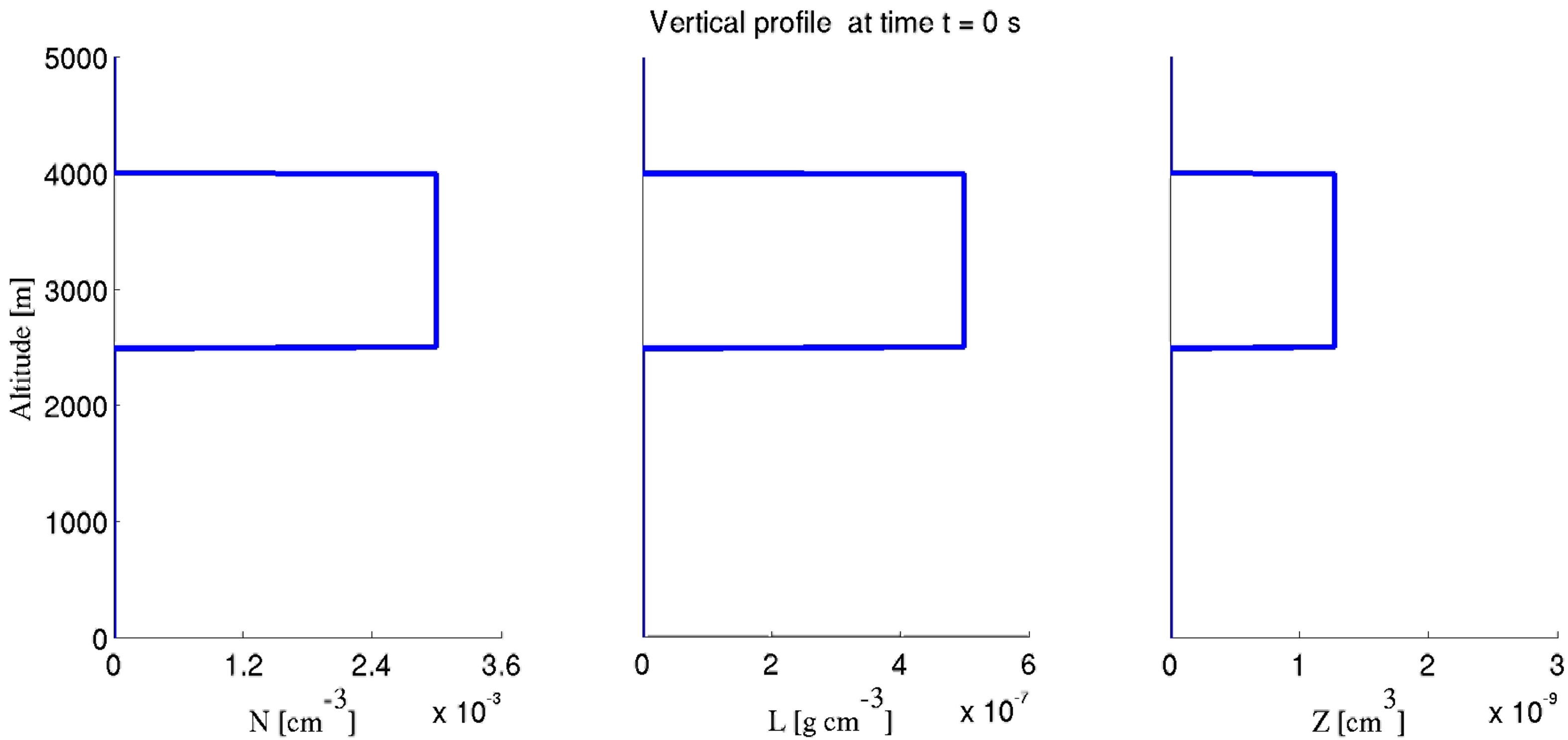
- Starting conditions :

$$f(D,z) = \begin{cases} \Gamma_{n_0,\gamma,\lambda}(D) = \dfrac{n_0 D^{\gamma-1} e^{-D/\lambda}}{\Gamma(\gamma)\lambda^{\gamma}}, & 2500m \leq z \leq 4000m \\ 0, & else \end{cases}$$

with n_0, γ and λ given parameters



Pure Sedimentation : comparison spectral vs QMoM



n_0= 3.10^3 m^{-3}, γ = 4 and λ ≈ 1,38.10^{-4}

solid : spectral *dashed : QMOM*

MoM relevance

t = 0 s

t = 100 s

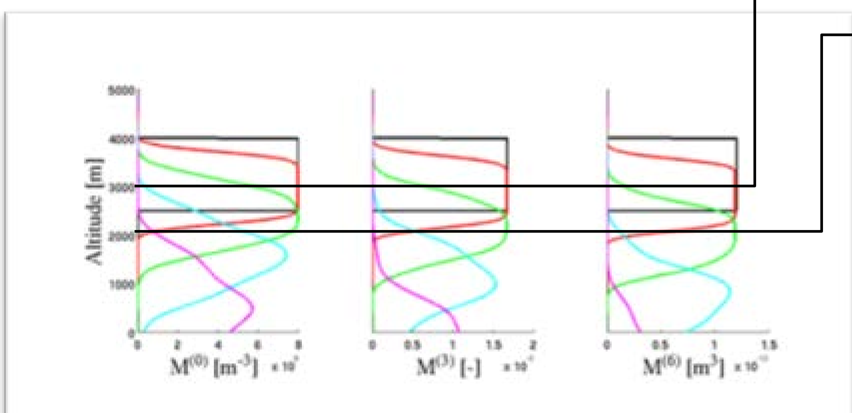
t = 300 s

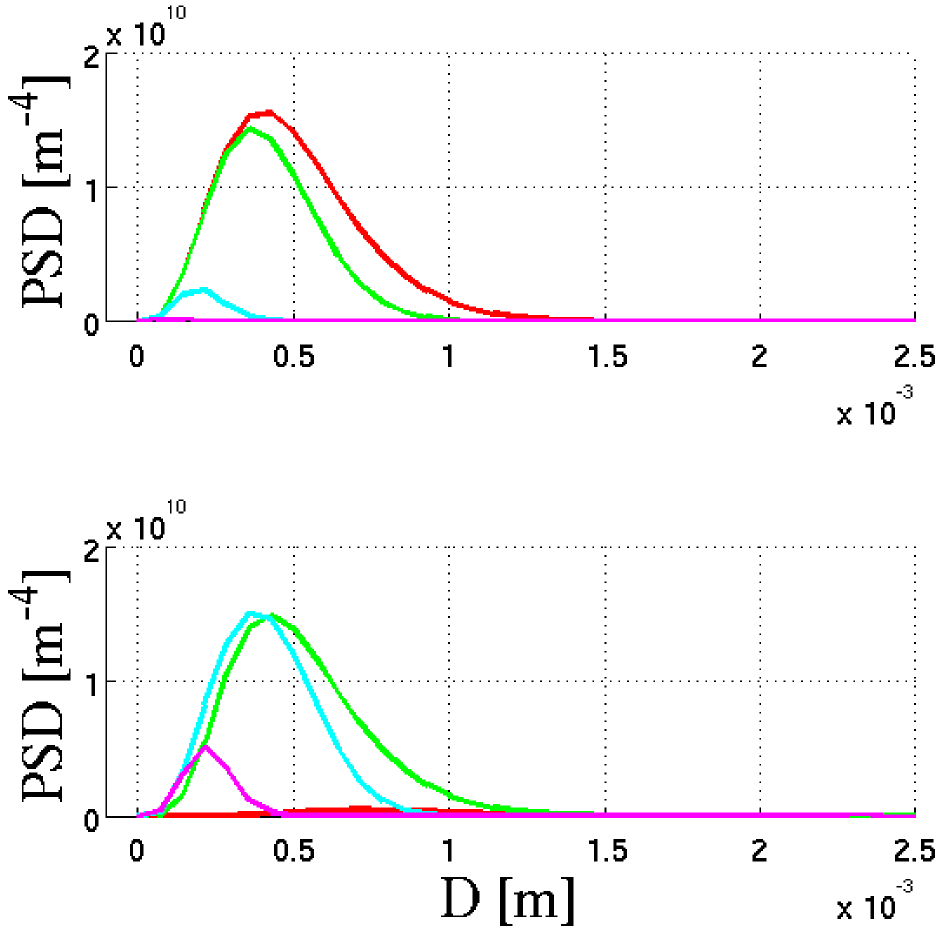
t = 600 s

t = 1000 s

z=3000m

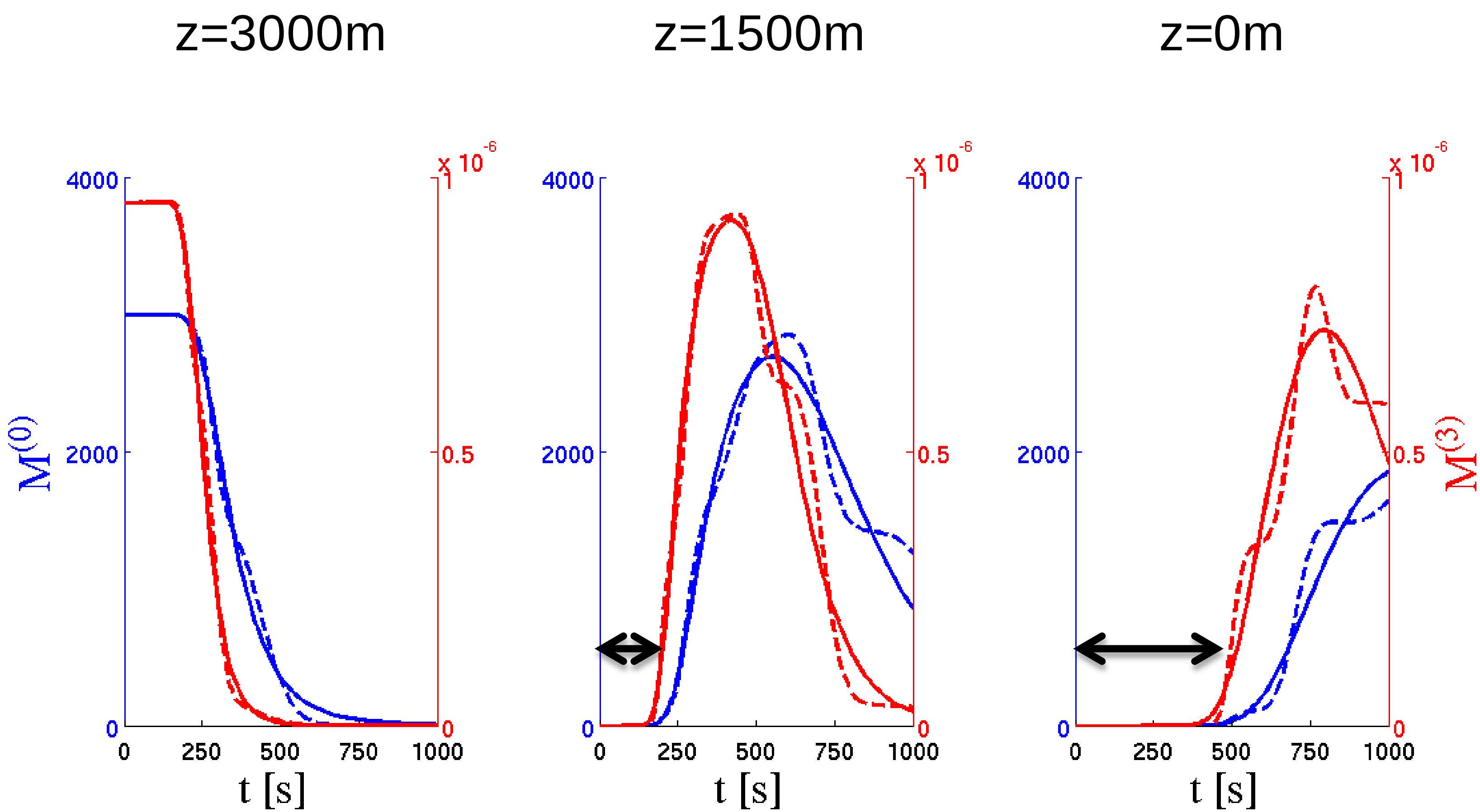
z=2000m





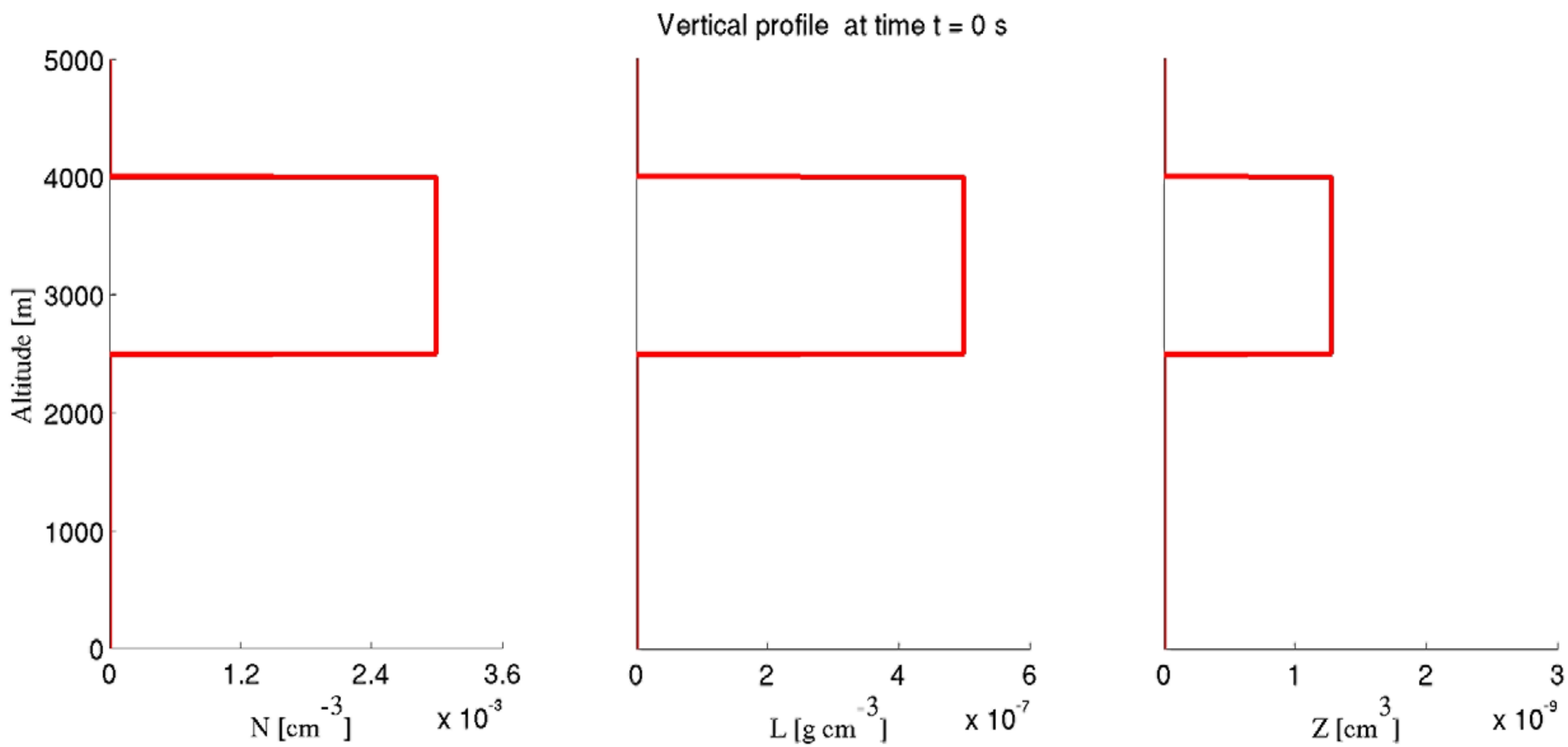
Evolution of Particle Size Distribution

Temporal evolution of the moments at different altitudes



solid : spectral *dashed : QMoM*

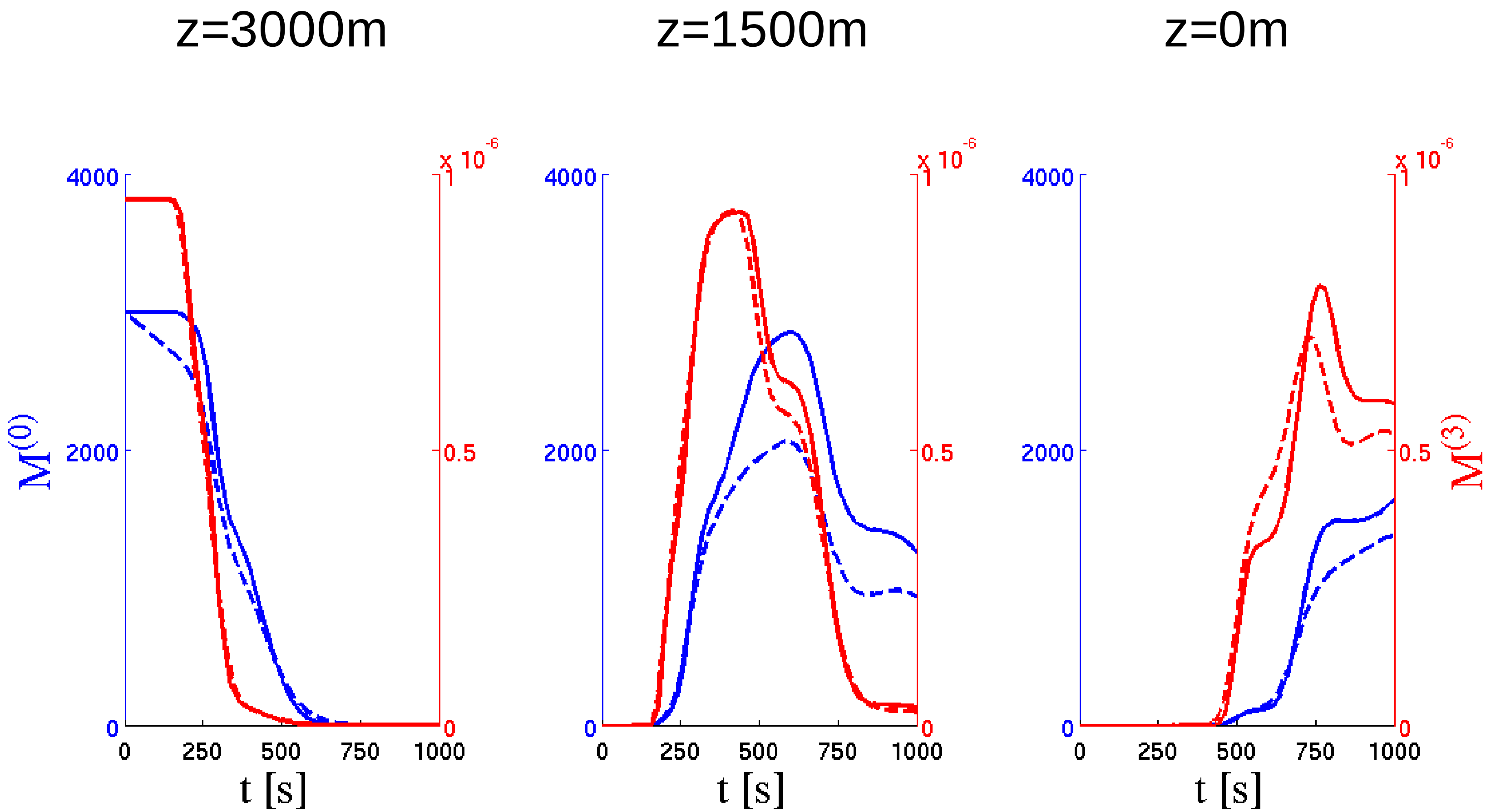
Comparison sedimentation without vs with coagulation



in red : without coagulation
in blue : with coagulation

(E_{eff} = 0.5)

Temporal evolution with and without coagulation



solid : without coagulation

dashed : with coagulation

Conclusions and Outlook

- Application of QMoM to 1D simple cases
  - QMoM deals efficiently with coagulation
  - Strong step towards study of more realistic cases

- Coming Work
  - LES modelling of a wind tunnel turbulent flow with liquid dispersed phase (Thévenin's group of University of Magdeburg)
  - Improvements and cross-checking with AWI (Bremerhaven) for 1D configurations and with KIT (Karlsruhe) for realistic coagulation kernels

Lehrstuhl für THERMODYNAMIK

International MetStröm-Conference 2011
09.06.2011, Berlin

Application of Quadrature Method of Moments for Sedimentation and Coagulation of Raindrops

Gary Jasor [1], Ulrike Wacker [2], Klaus Beheng [3] and Wolfgang Polifke [1]

[1] TU München [2] AWI Bremerhaven [3] KIT Karlsruhe

With financial Support of Priority Programm SPP1276 Metström (DFG)
Project: *Multi-scale Modelling of the Population Dynamics of Hydrometeors with Methods of Moments (MoM's)*