Numerical studies of conditionally unstable moist convection

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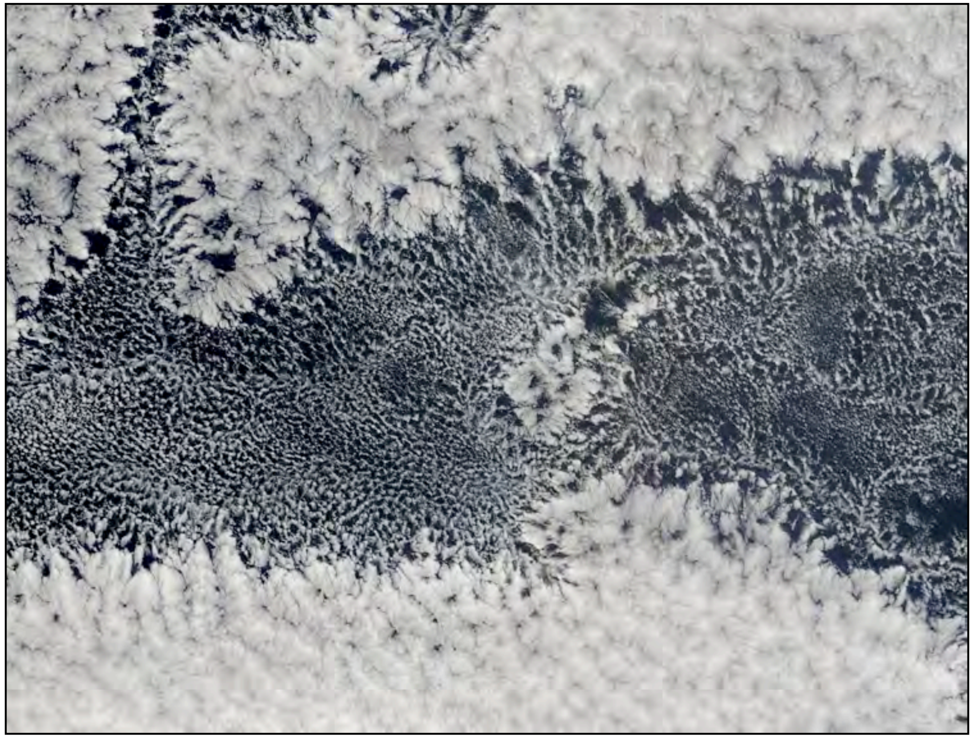


DNS of moist convection

Bretherton, J. Atmos. Sci. 1987 & 1988; Pauluis & JS, Comm. Math. Sci. 2010

What is the least set of moist convection equations to describe shallow cloud formation processes?

- No ice
- Boussinesq approximation
- Local thermodynamic equilibrium

$$q_T = q_l + q_v$$

- Piecewise linear equation of state

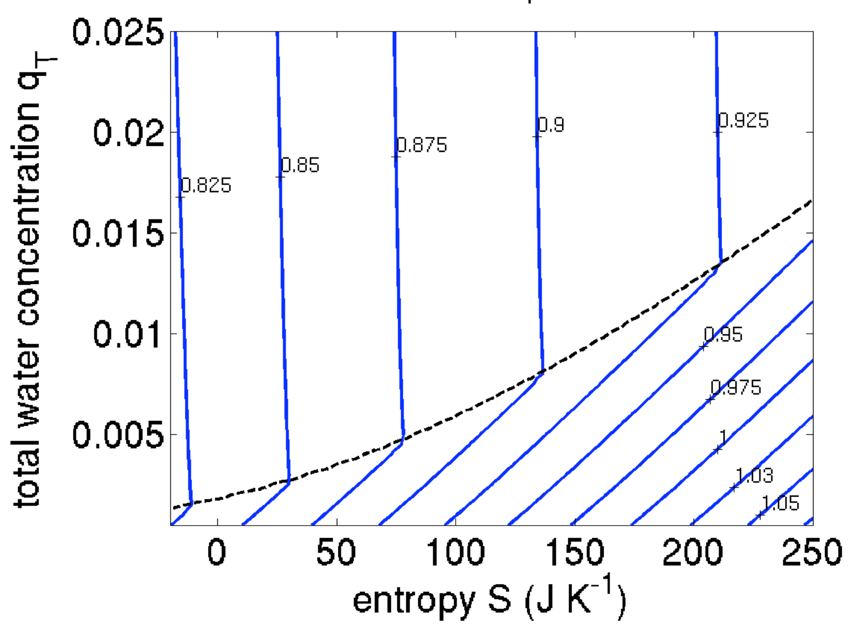
$$(T, q_v, q_l) \rightarrow (D, M)$$

- Saturation condition

$$\vec{f} = \max(M, D - N_s^2 z)\vec{e}_z$$

Brunt-Vaisala frequency N_s

Specific volume $\alpha(S,q_T,p)$ for p = 900mb



Shallow moist RB convection

$$\nabla \cdot \vec{u} = 0$$

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla)\vec{u} = -\frac{1}{\rho_0}\nabla p + \nu \nabla^2 \vec{u} + B(D, M, z)\vec{e}_z$$

RB convection

$$\frac{\partial D'}{\partial t} + (\vec{u} \cdot \nabla)D' = \kappa \nabla^2 D' + \frac{D_0 - D_H}{H} u_z$$

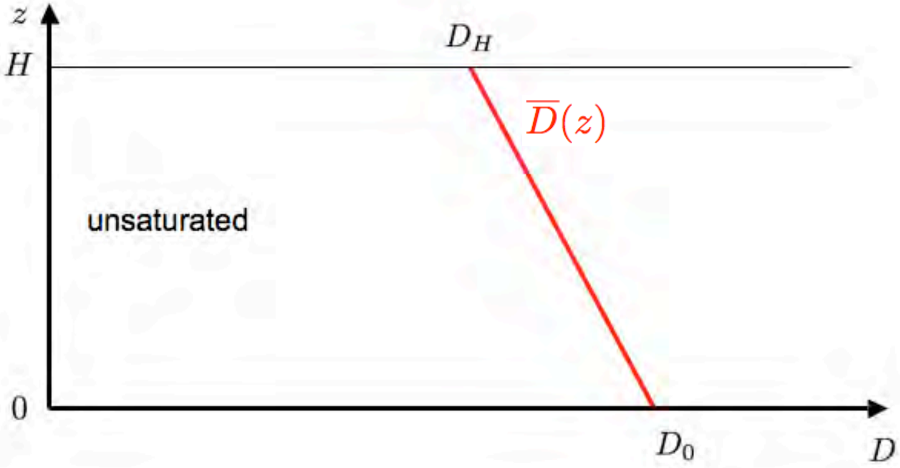
$$B(D) = D$$

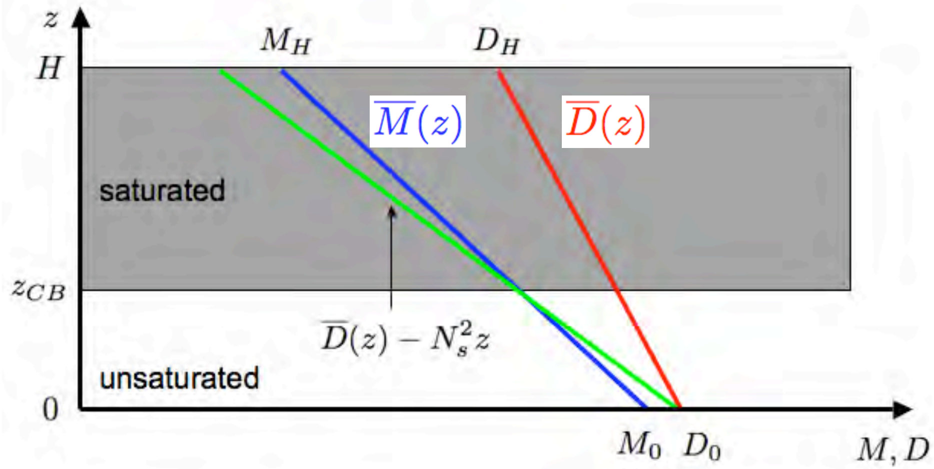
Moist RB convection

$$\frac{\partial D'}{\partial t} + (\vec{u} \cdot \nabla)D' = \kappa \nabla^2 D' + \frac{D_0 - D_H}{H} u_z$$

$$\frac{\partial M'}{\partial t} + (\vec{u} \cdot \nabla)M' = \kappa \nabla^2 M' + \frac{M_0 - M_H}{H} u_z$$

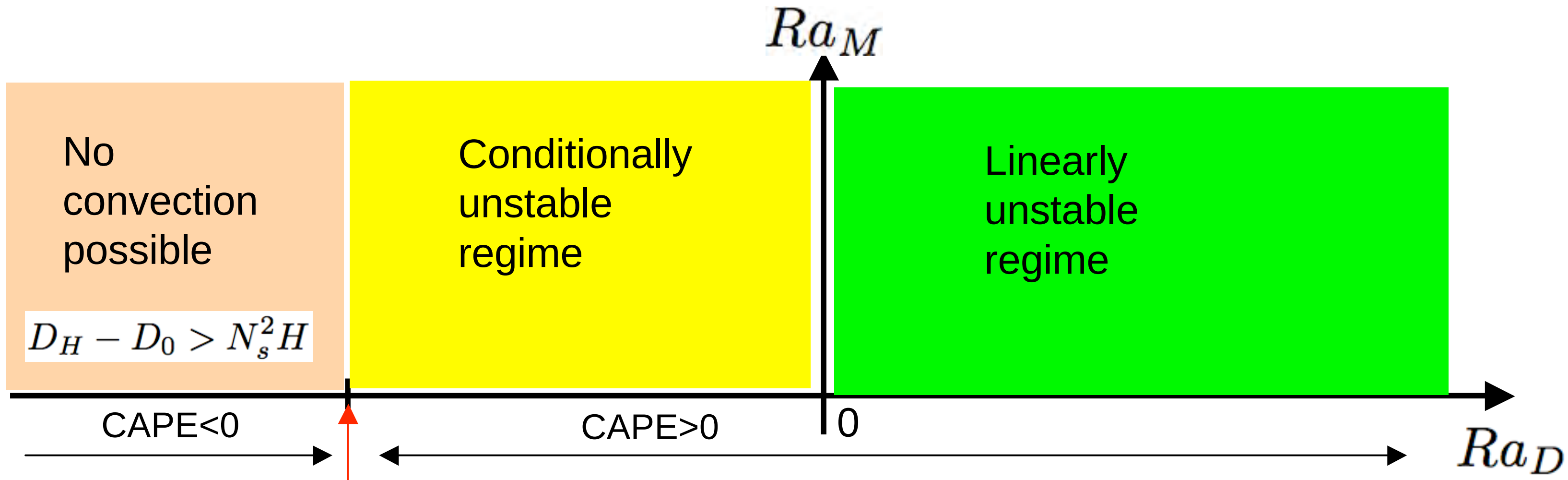
$$B(D, M, z) = \max(M, D - N_s^2 z)$$





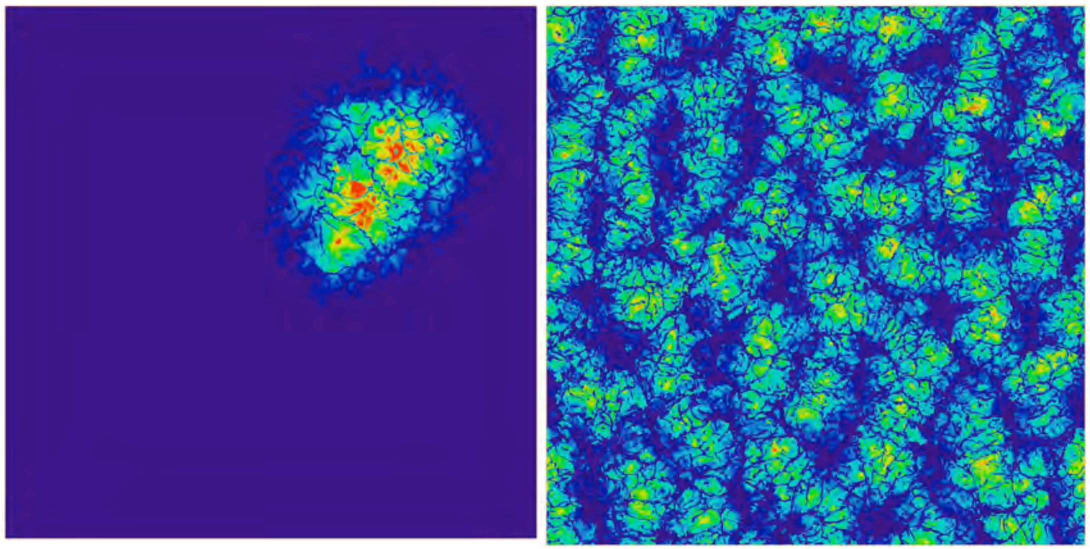
Free-slip boundary conditions

Linearly & conditionally unstable equilibrium



$$-Ra_{BV} = -\frac{N_s^2 H^4}{\nu\kappa}$$

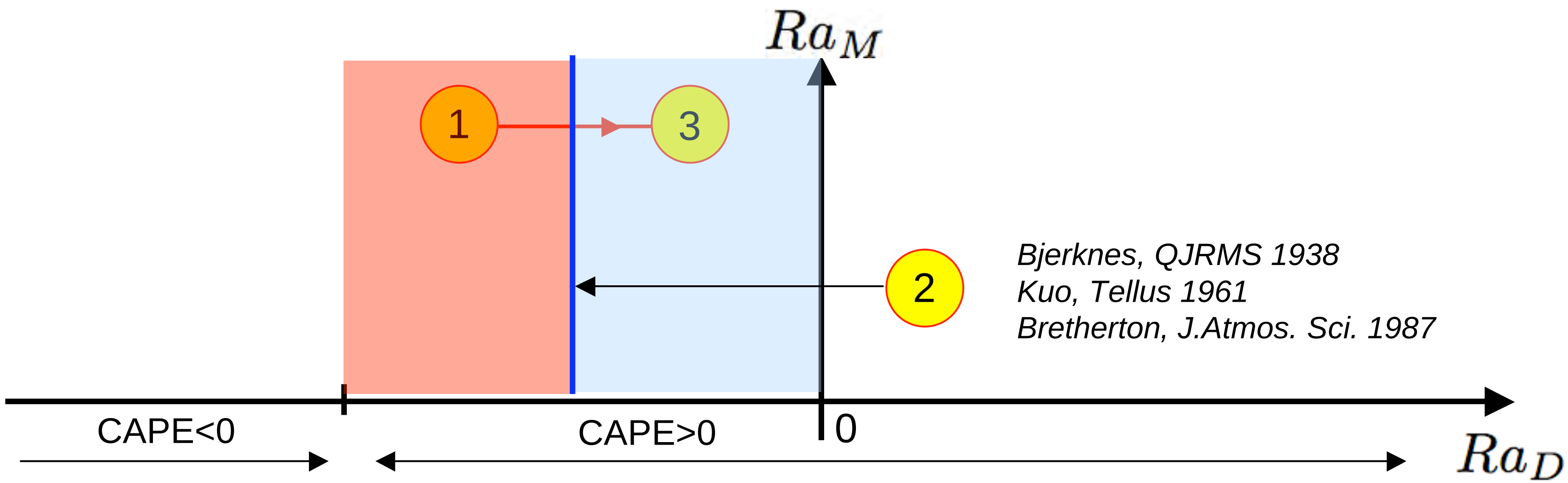
$Ra_D < 0 \quad Ra_M > 0$ $\qquad$ $Ra_D > 0 \quad Ra_M > 0$



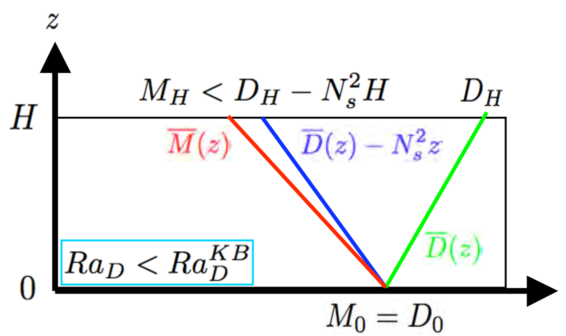
Localized vs. spacefilling turbulence

Weidauer, Pauluis & JS, New J. Phys. 2010
JS & Pauluis, J. Fluid Mech. 2010

Two classes of conditionally unstable regimes

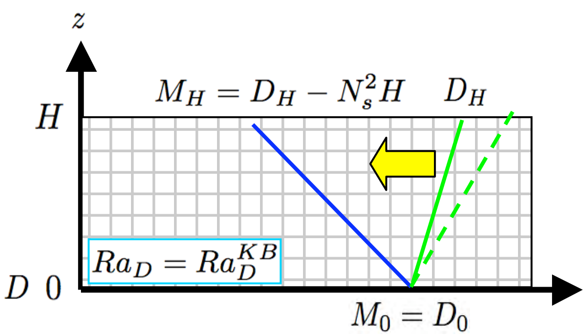


Bjerknes, QJRMS 1938
Kuo, Tellus 1961
Bretherton, J.Atmos. Sci. 1987



1

Subcritical case



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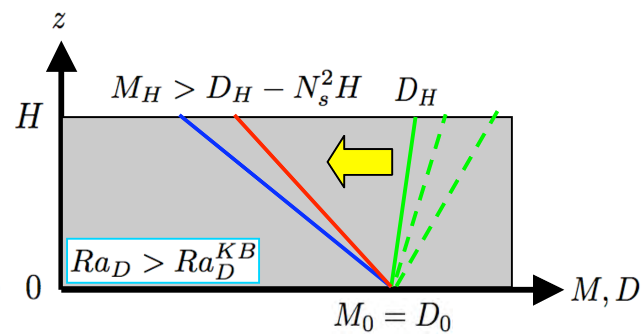
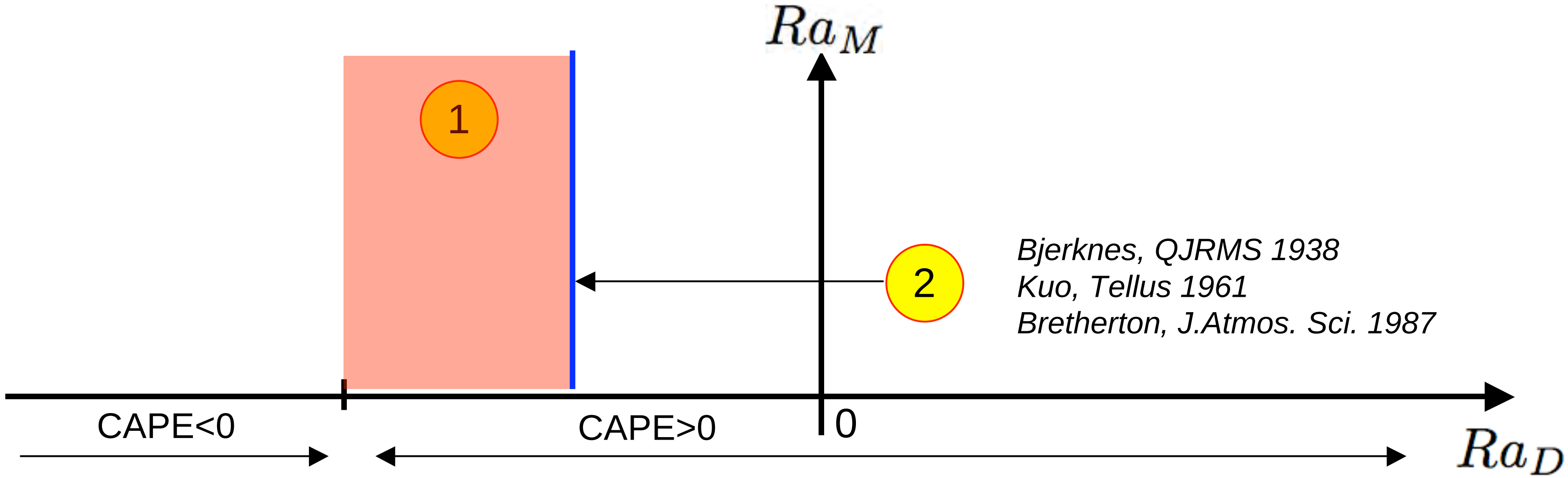
Kuo-Bretherton case



3

Supercritical case

Conditionally unstable regimes



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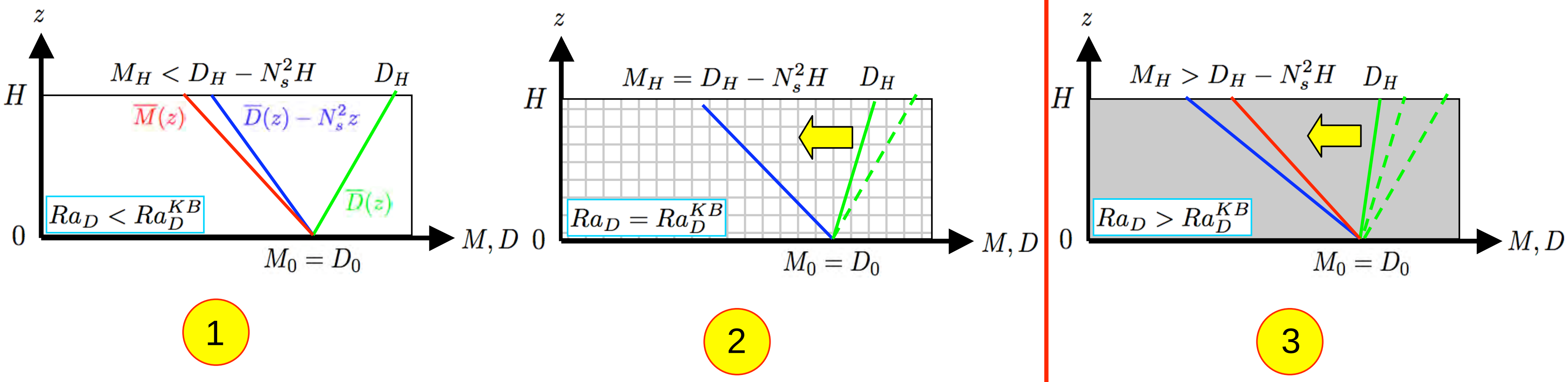


Subcritical case

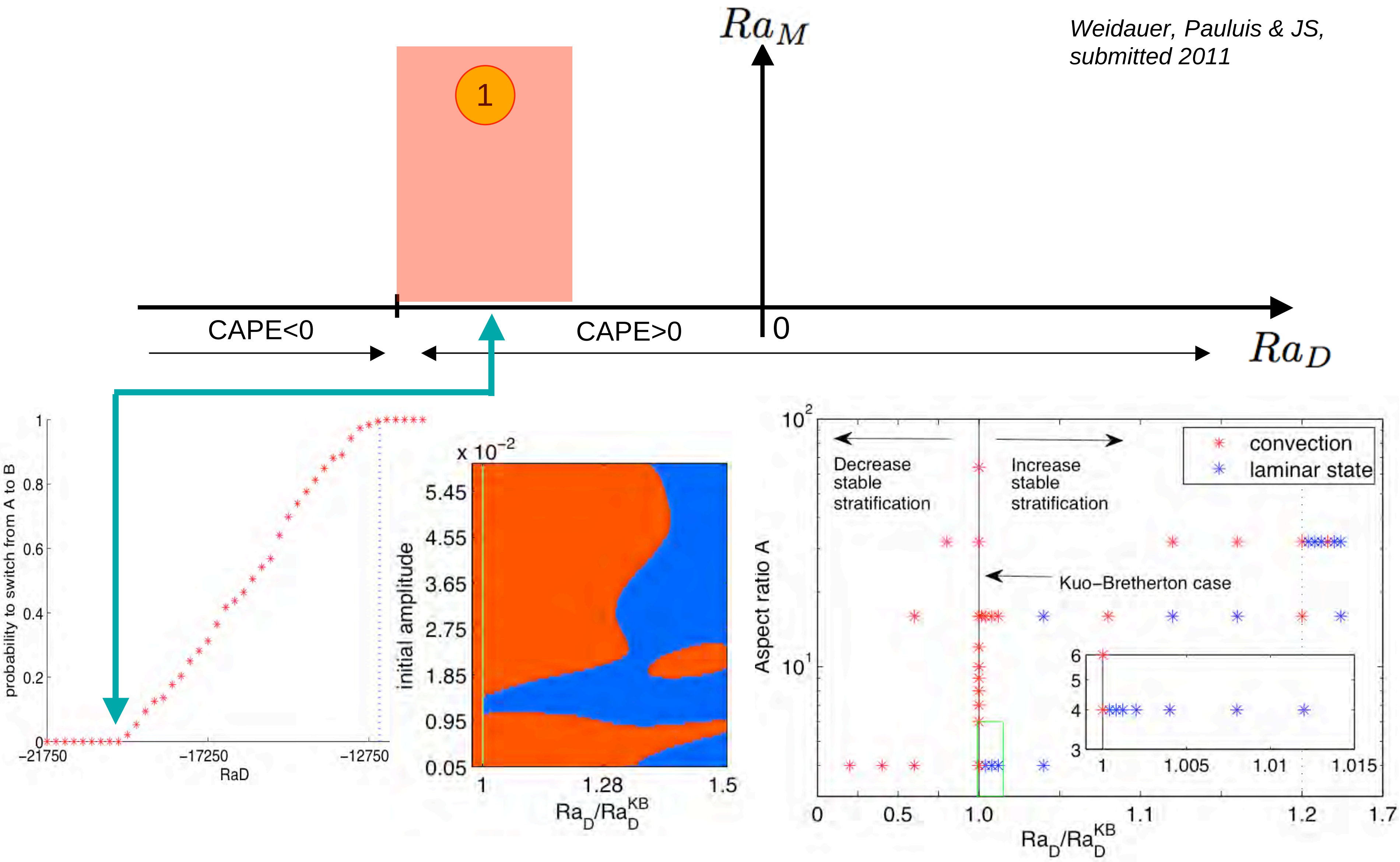
Kuo-Bretherton case

Supercritical case

Focus

Transition to moist convection

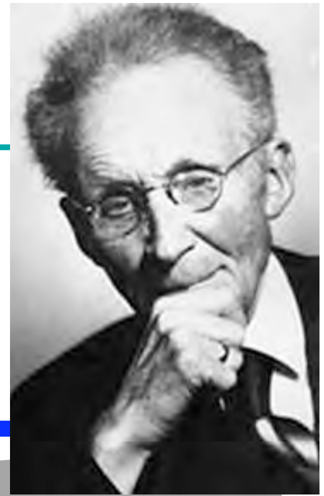
Weidauer, Pauluis & JS, submitted 2011



Galerkin model at lower Ra

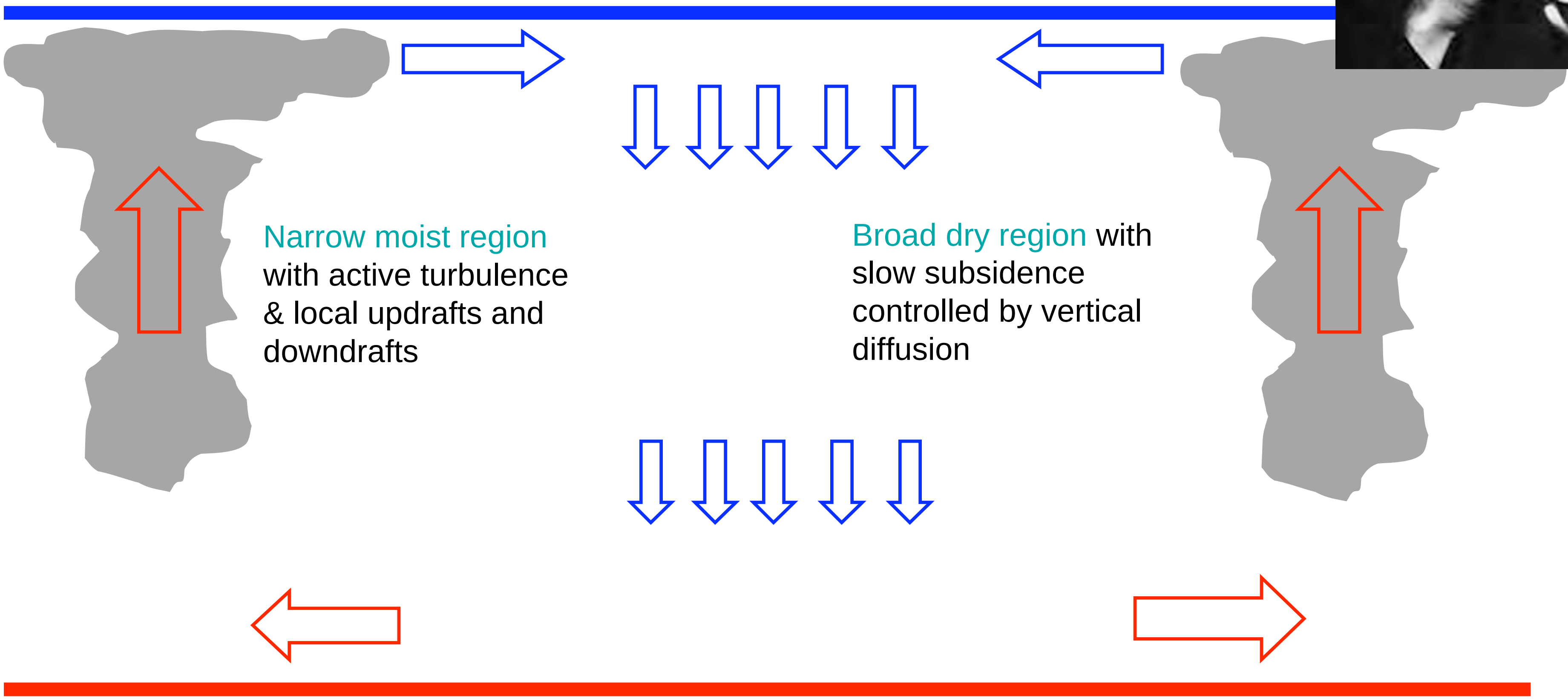
DNS at higher Ra

Which cloud patterns are expected?



Bjerknes, Quat. J. Royal Meteor. Soc. 1938

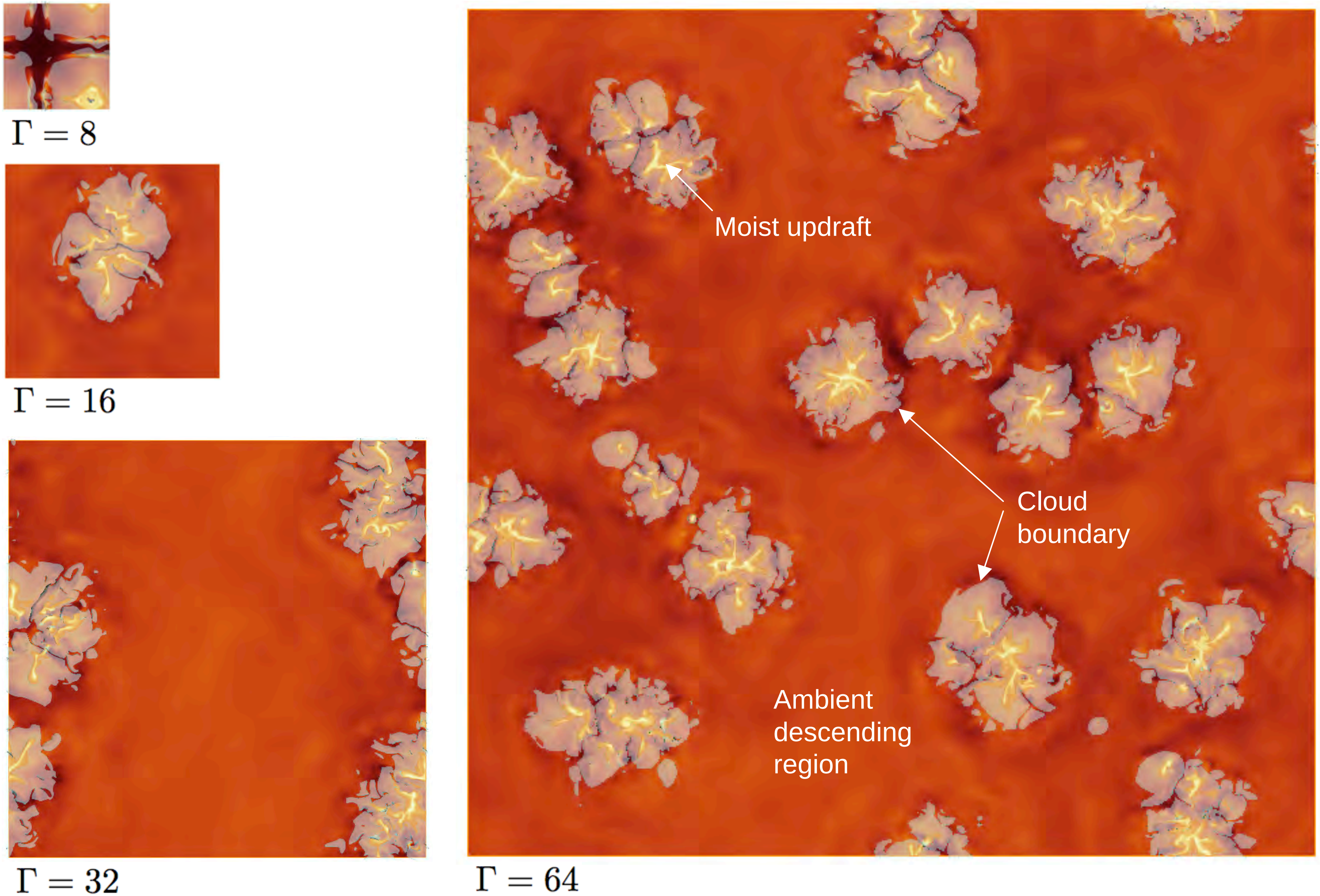
Entrainment and drying above cloud aggregate



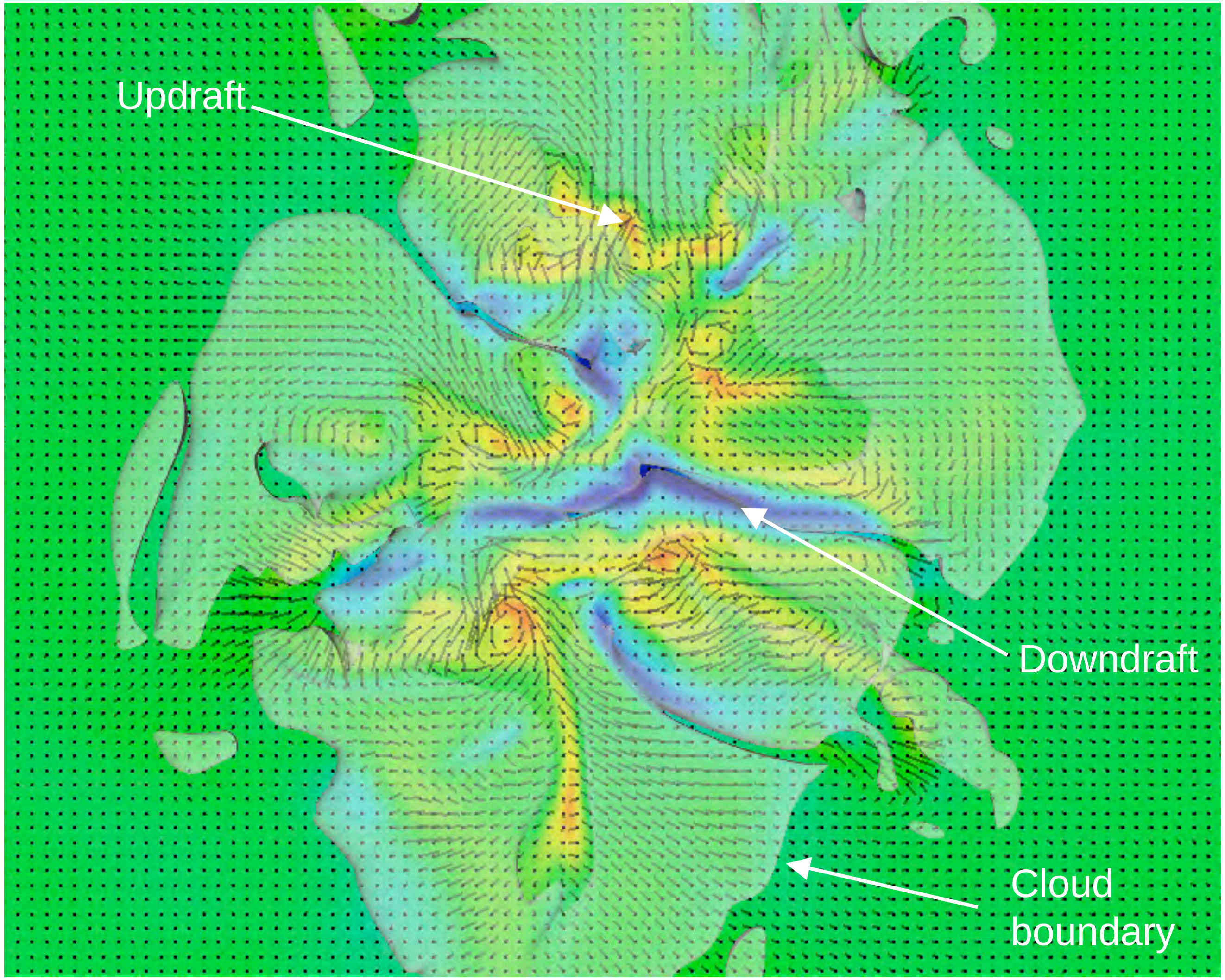
Enhanced evaporation in the dry region

Cloud formation in moist RB convection

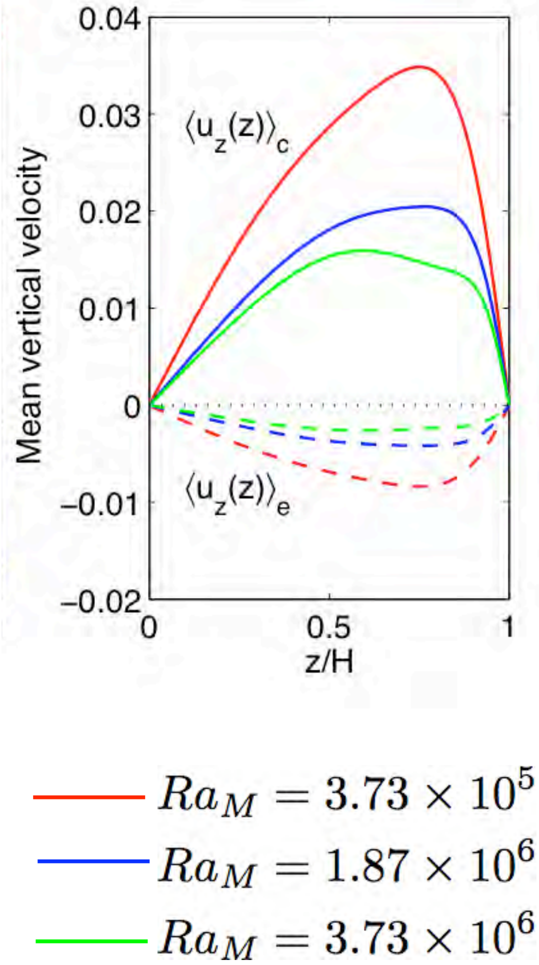
Pauluis & JS, submitted 2011



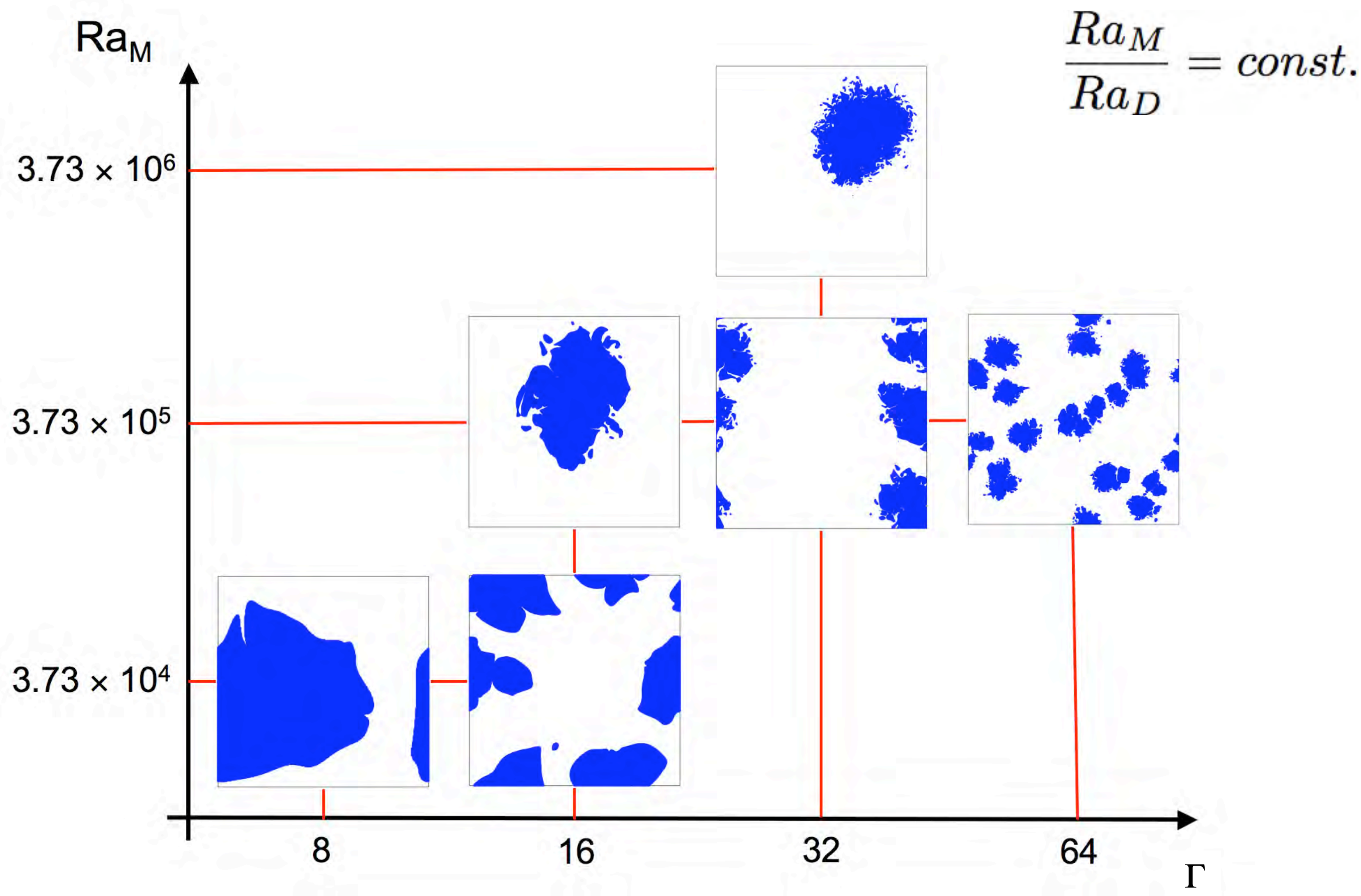
Turbulent velocity inside cloud aggregate



$\Gamma = 16$

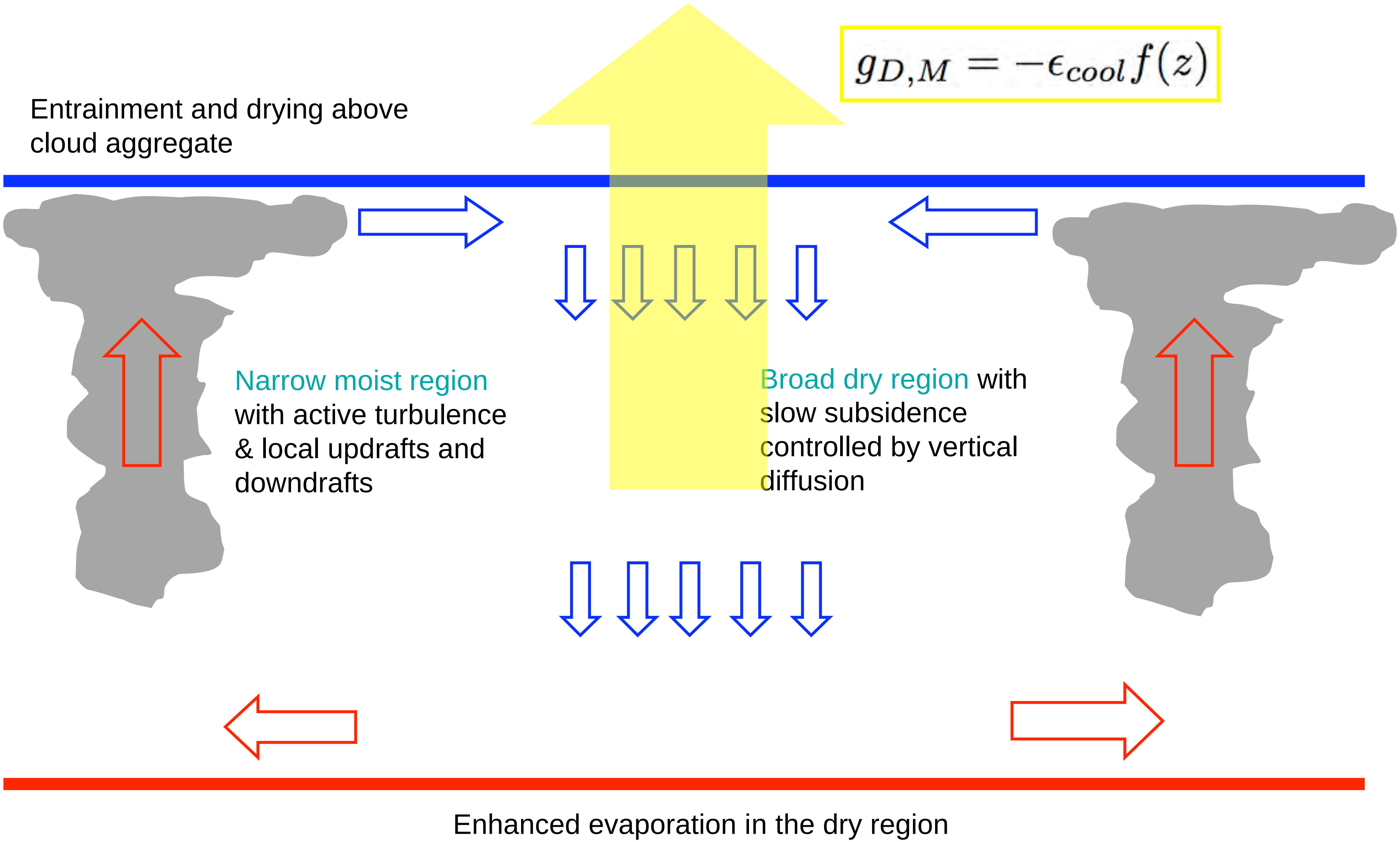


Trends with Rayleigh number

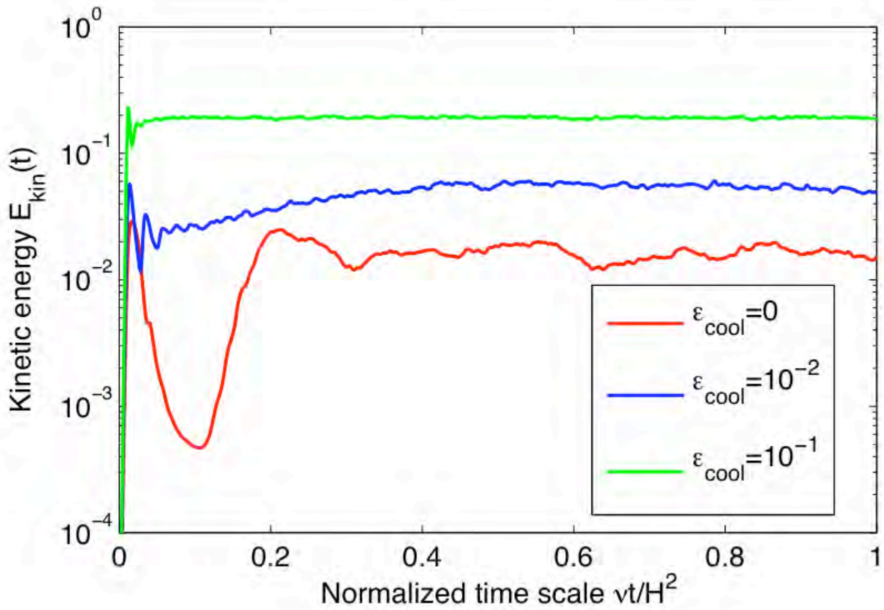


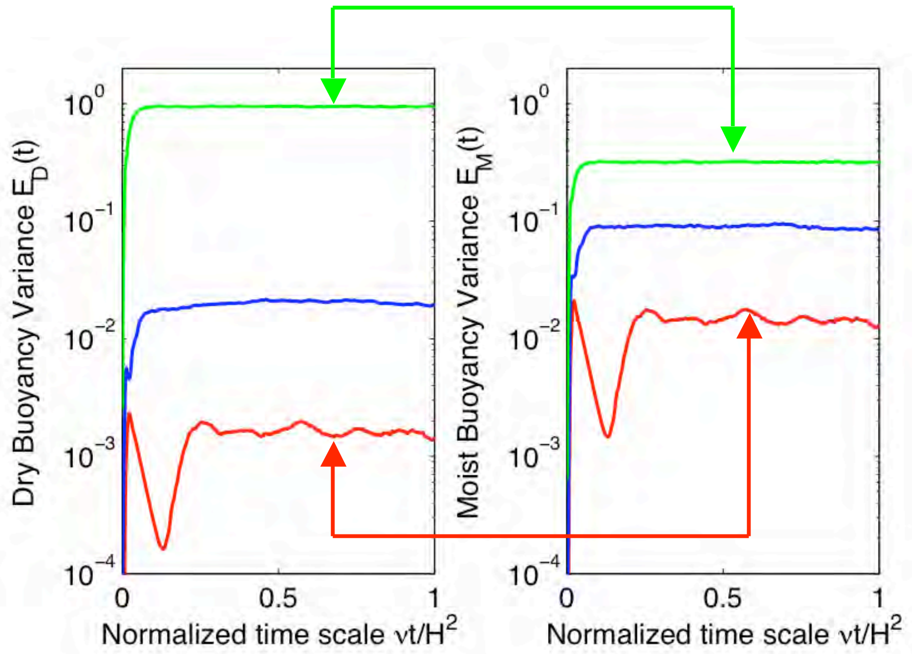
Ever larger environments for dry subsidence required

Radiative cooling

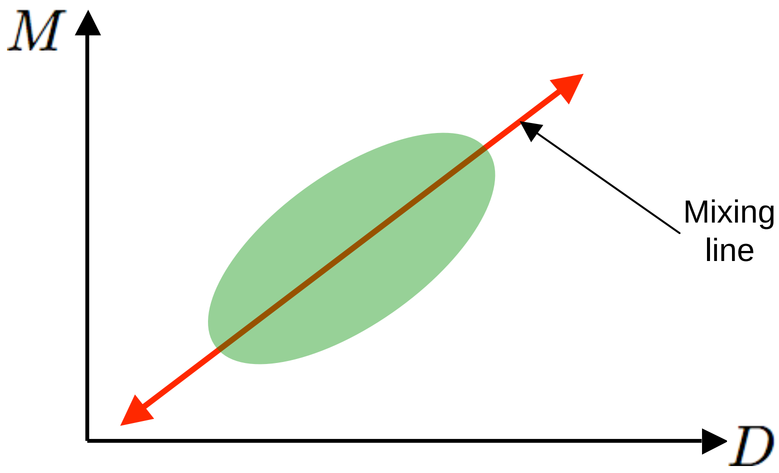


Re-amplification of turbulence



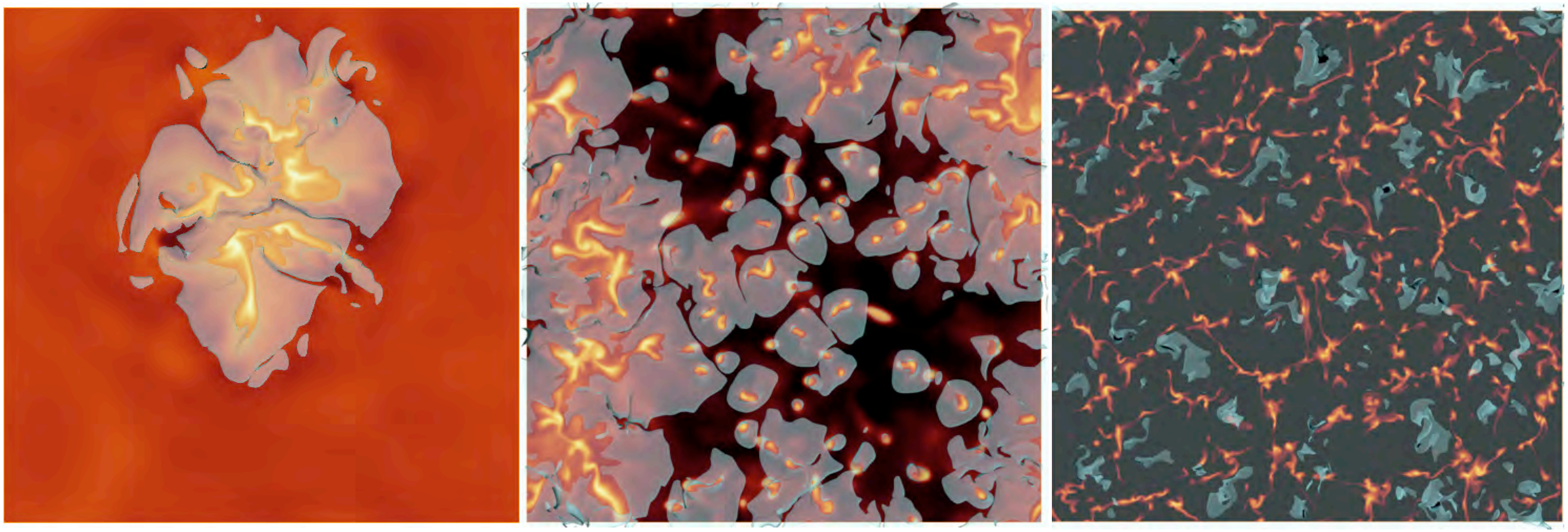


Radiative cooling breaks the synchronicity between dry and moist buoyancy & re-amplifies conditionally unstable convection



Cloud aggregation

$\epsilon_{cool} = 0$ $\epsilon_{cool} = 10^{-2}$ $\epsilon_{cool} = 10^{-1}$



Localized cloud ⟶ Closed cloud layer

Summary

- Moist Boussinesq model with simplified condensation scheme describes different regimes of moist convection
- Systematic statistical analysis accessible for DNS in large-aspect ratio domains
- Convection is highly asymmetric (up-down) & transition to convection depends on aspect ratio
- Formation of localized cloud aggregates for sufficiently large aspect ratio
- Radiative cooling re-amplifies convective turbulence & alters the cloud cover drastically

http://www.tu-ilmenau.de/tsm

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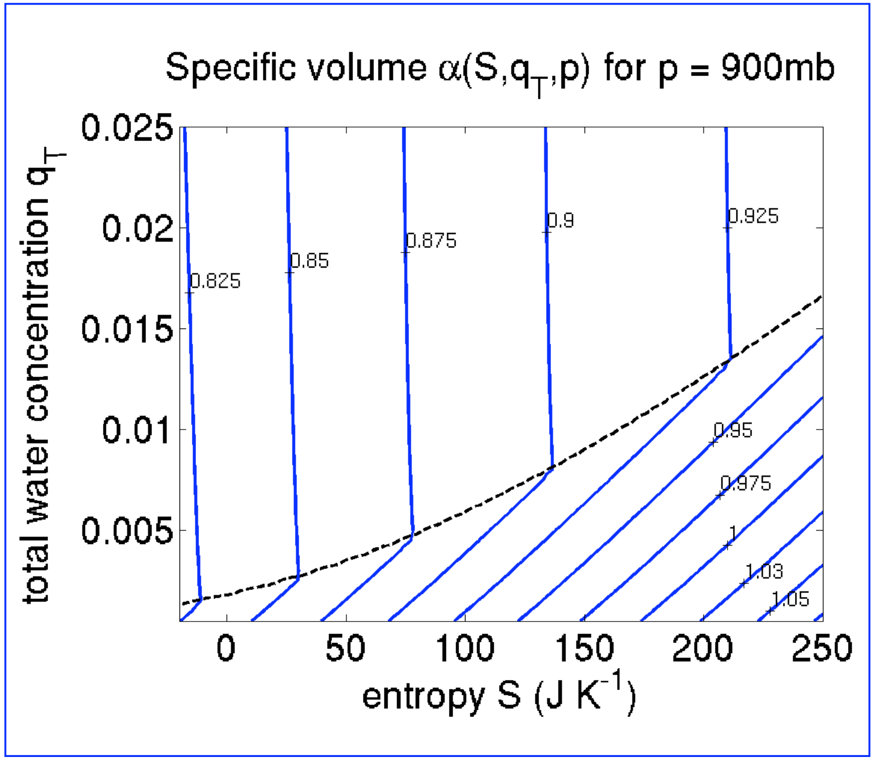
Piecewise linear thermodynamics

Pauluis & JS, Comm. Math. Sci. 2010

$$B(S, q_T, z)\, \vec{e}_z$$



Linear expansion on both sides of phase boundary

$$\left.\frac{\partial B}{\partial S}\right|_{q_T,z} = \begin{cases} B_{S,u} & \text{if } q_T \le q_{sat}(S,z) \\ B_{S,s} & \text{if } q_T > q_{sat}(S,z) \end{cases}$$

$$\left.\frac{\partial B}{\partial q_T}\right|_{S,z} = \begin{cases} B_{q_T,u} & \text{if } q_T \le q_{sat}(S,z) \\ B_{q_T,s} & \text{if } q_T > q_{sat}(S,z)\,. \end{cases}$$

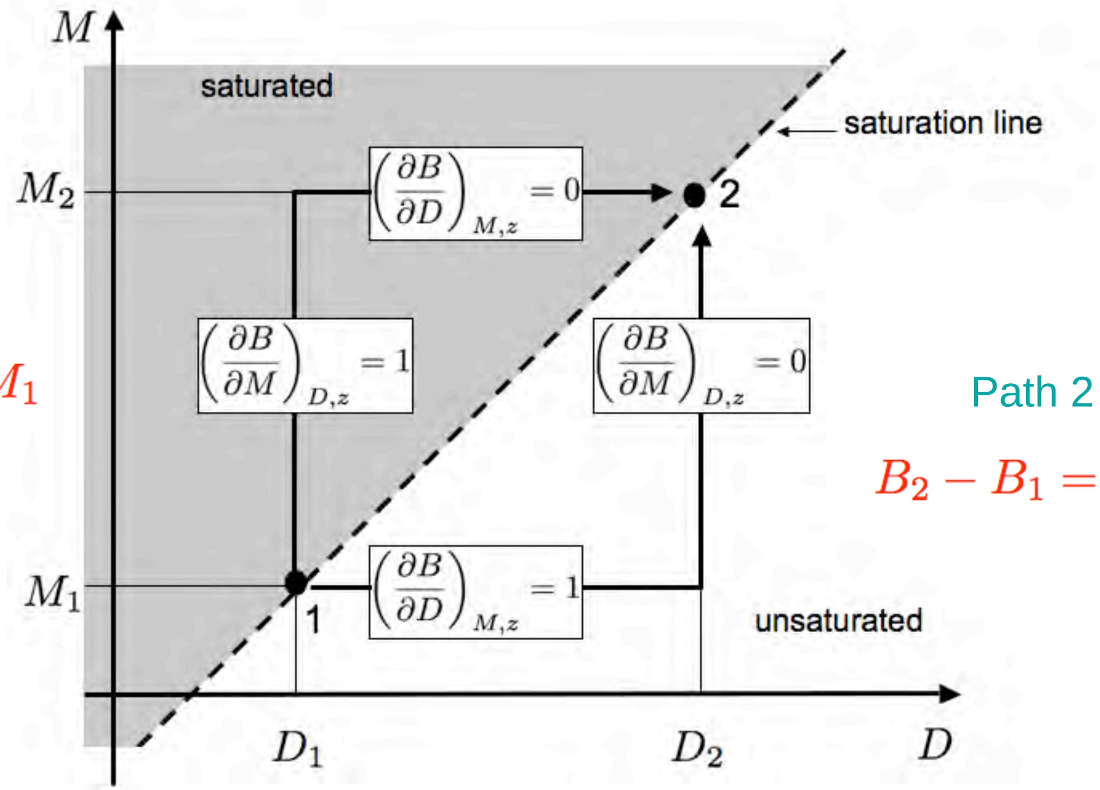
Linear combination to new variables

$$D = B_{S,u}(S - S_{ref}) + B_{q_T,u}(q_T - q_{T,ref})$$
$$M = B_{S,s}(S - S_{ref}) + B_{q_T,s}(q_T - q_{T,ref})$$

Explicit saturation condition

Path 1

$$B_2 - B_1 = M_2 - M_1$$



Path 2

$$B_2 - B_1 = D_2 - D_1$$

Saturation condition $F(M, D, z) \geq 0$

$$\rightarrow \quad M_2 - M_1 = D_2 - D_1$$

$$\rightarrow \quad M - D = f(z)$$ with $$f(z) = -N_s^2 z$$

N_s is a Brunt-Vaisala fequency given by $N_s^2 = \frac{g}{T_{ref}}(\Gamma_d - \Gamma_s)$