

New Results about the Cloud-Top Entrainment Instability



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International MetStröm Conference, FU Berlin
6-10 June 2011

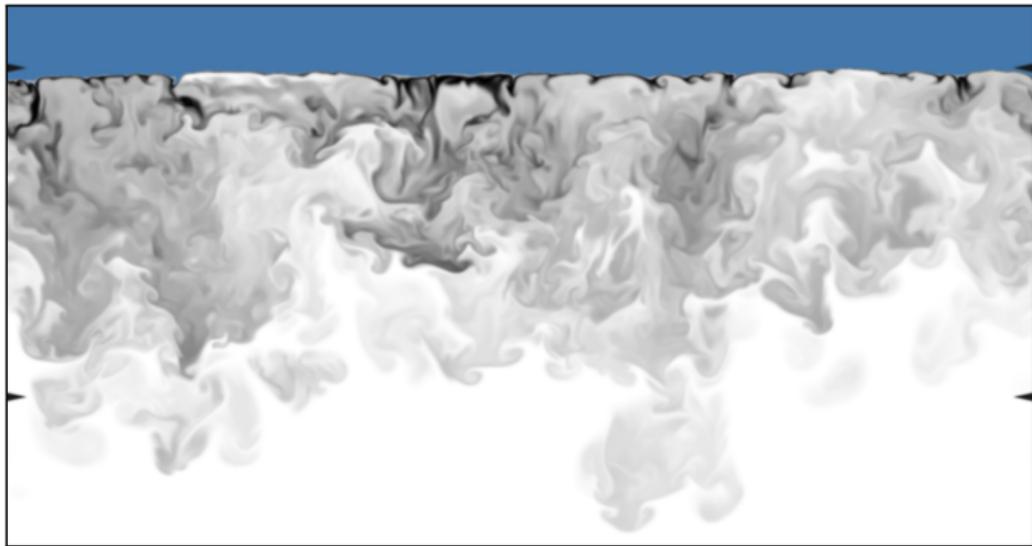
Outline

The Cloud-Top Entrainment Instability

New Results from Direct Numerical Simulations

Evaporative Cooling and Clouds

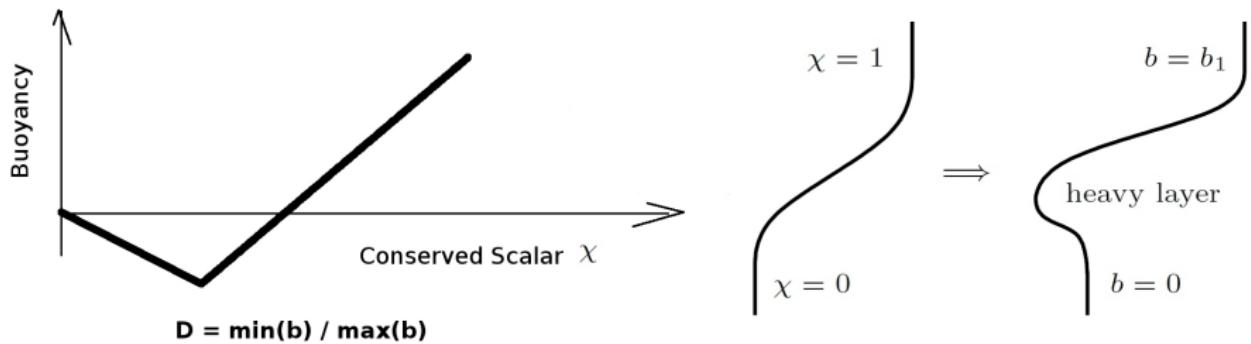
Evaporation at cloud/free-atmosphere interface → cooling → convection:



Buoyancy Reversal

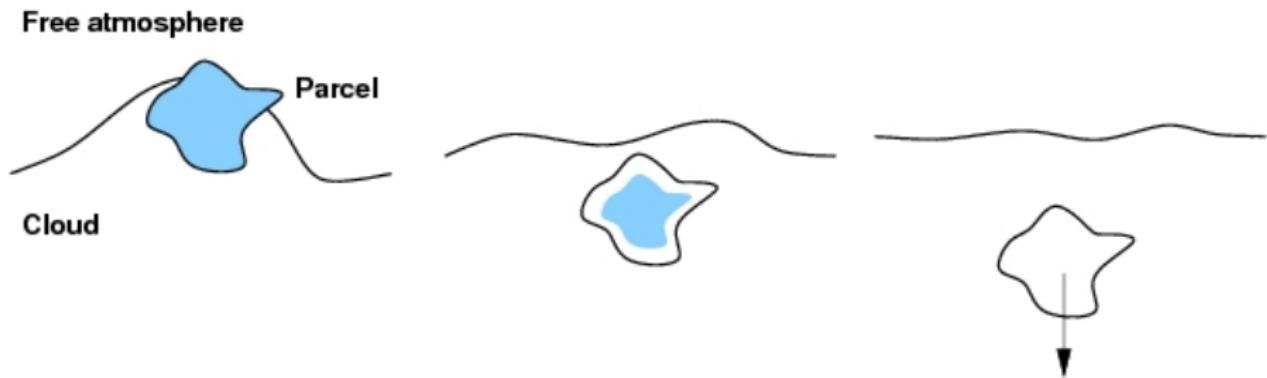
The buoyancy may not follow a linear relation wrt conserved scalars:

sign change \Rightarrow buoyancy reversal



Stratocumulus Top

In addition, inversion at the cloud boundary:



adapted from D. Randall, *J. Atmos. Sci.*, 1980

Cloud-Top Entrainment Instability

(Randall, 1980; Deardorff, 1980)



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The Cloud-Top Entrainment Instability

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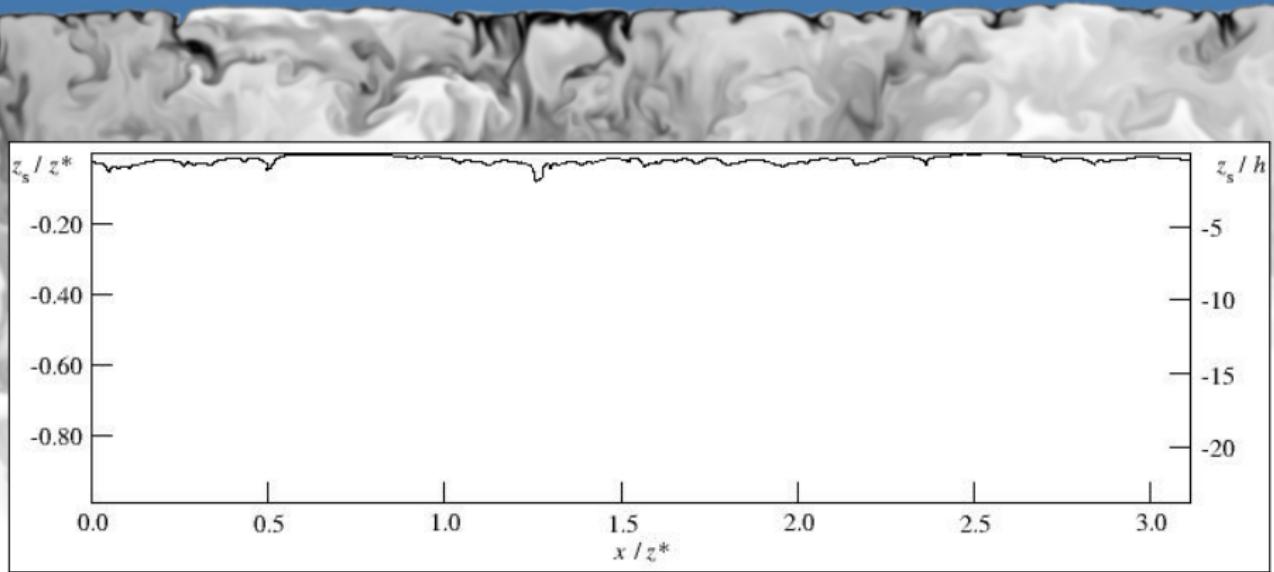
DNS, grid size $2048 \times 2048 \times 1536$ (JPM, 2010; JPM et al., 2010)



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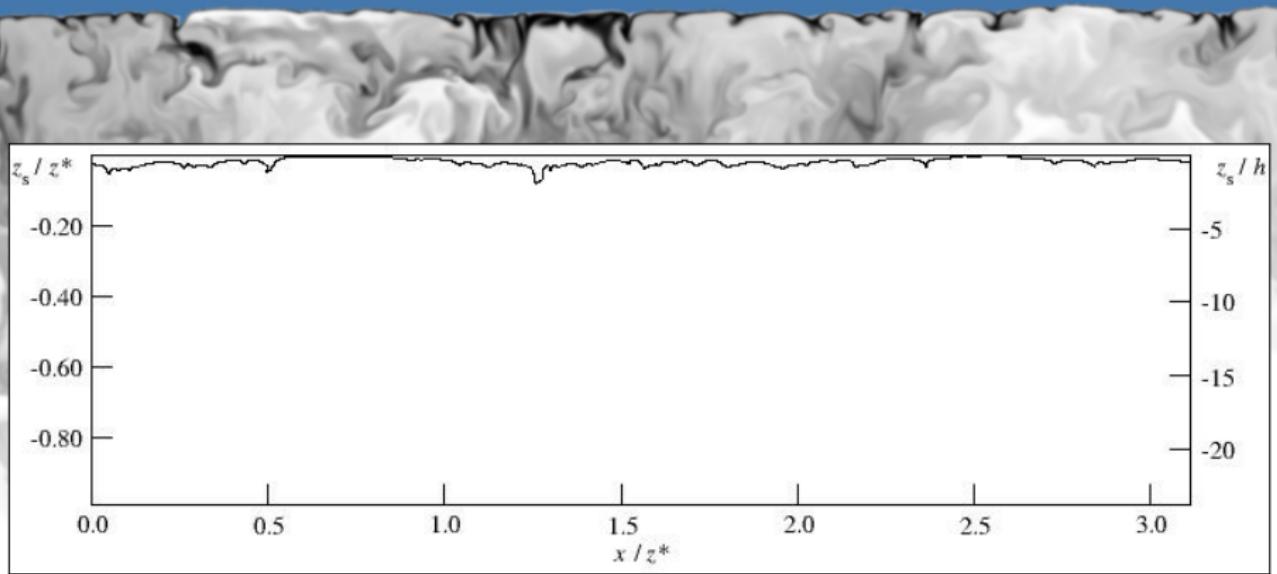


New Results from Direct Numerical Simulations



Instantaneous cloud-top $\{x : \chi(x, t) = \chi_s\}$

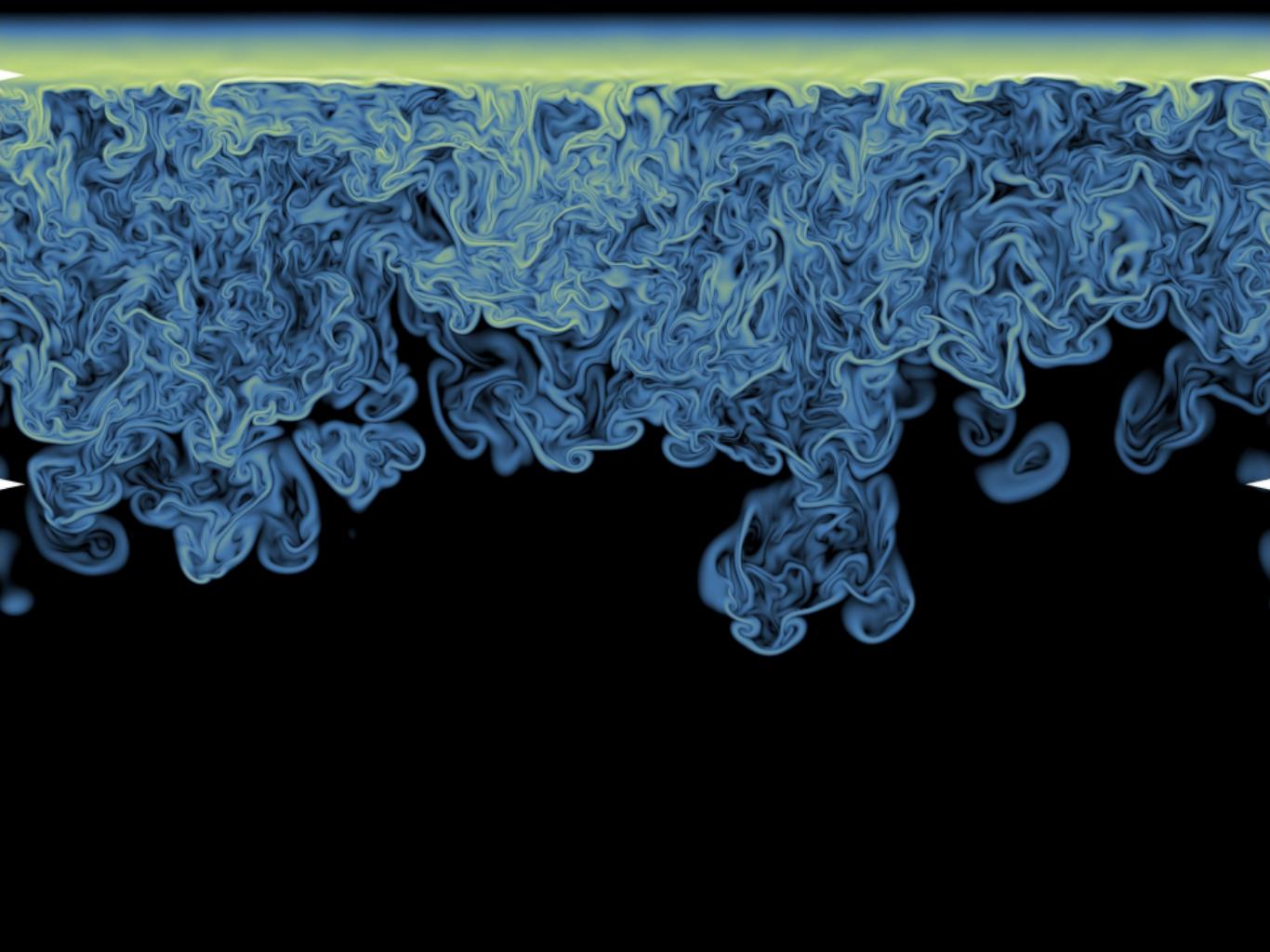




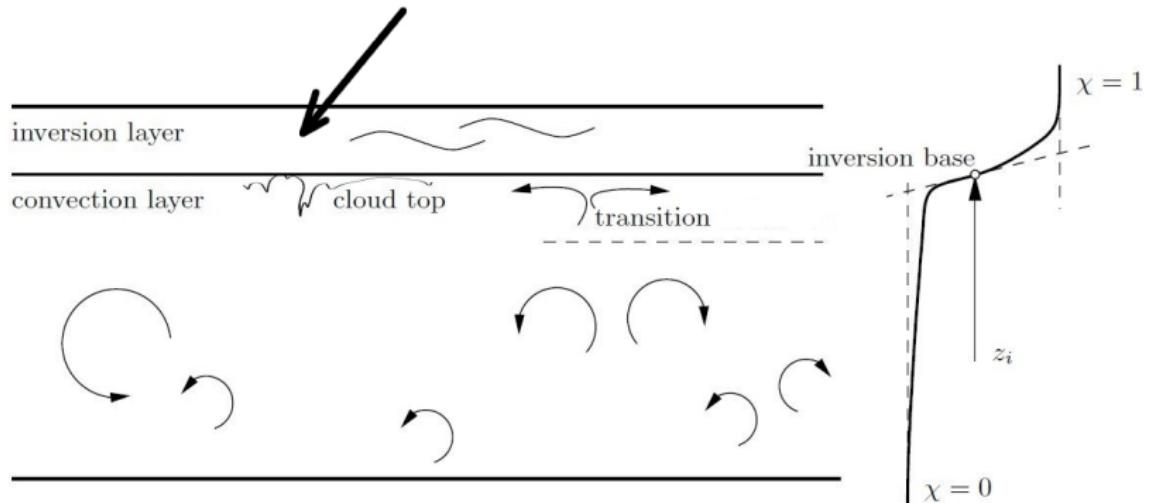
Instantaneous cloud-top $\{x : \chi(x, t) = \chi_s\}$

Cloud-top not broken and flat:

Two time scales associated w/ buoyancies b_1 , $|b_s|$: $D = |b_s|/b_1 \ll 1$.



Inversion Layer



Inversion Layer

Molecular transport controls the system



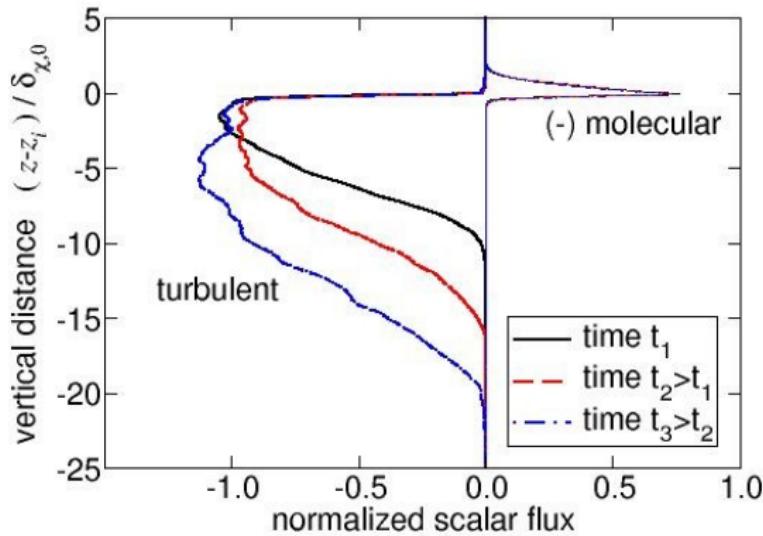
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New Results from Direct Numerical Simulations

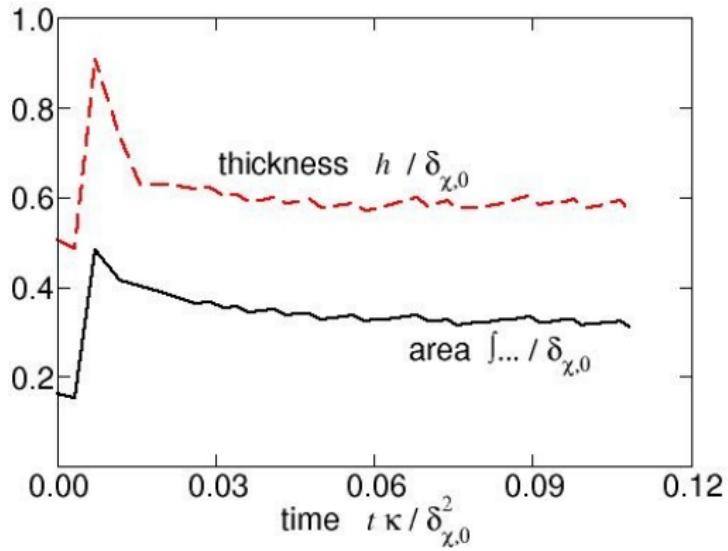
Inversion Layer

$$\frac{d}{dt} \left[\int_{z_i}^{\infty} (1 - \langle \chi \rangle) dz \right] + \langle w' \chi' \rangle(z_i, t) = \kappa \frac{\partial \langle \chi \rangle}{\partial z}(z_i, t) - (1 - \chi_i) \frac{dz_i}{dt}$$



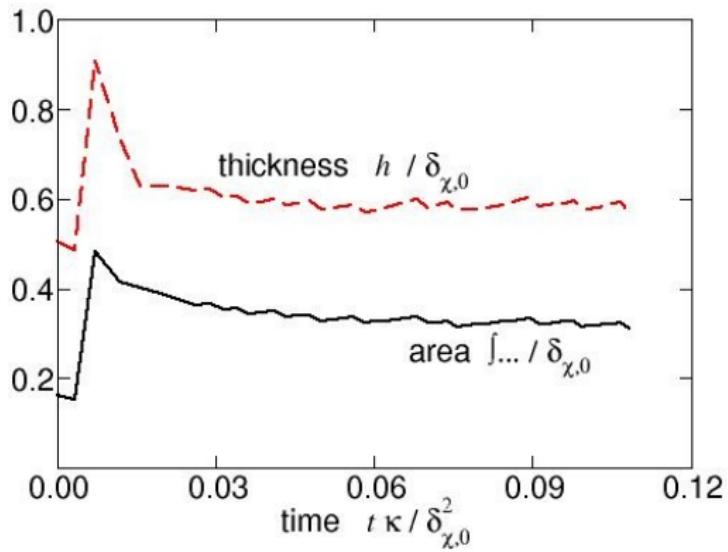
Inversion Layer

$$\frac{d}{dt} \left[\int_{z_i}^{\infty} (1 - \langle \chi \rangle) dz \right] + \cancel{\langle w' \chi' \rangle(z_i, t)} = \kappa \frac{\partial \langle \chi \rangle}{\partial z}(z_i, t) - (1 - \chi_i) \frac{dz_i}{dt}$$



Inversion Layer

$$\frac{d}{dt} \left[\int_{z_i}^{\infty} (1 - \langle \chi \rangle) dz \right] + \cancel{\langle w' \chi' \rangle(z_i, t)} = \kappa \frac{\partial \langle \chi \rangle}{\partial z}(z_i, t) - (1 - \chi_i) \frac{dz_i}{dt}$$



Dominant balance

$$w_e = \frac{dz_i}{dt} \simeq \frac{\kappa}{h}$$

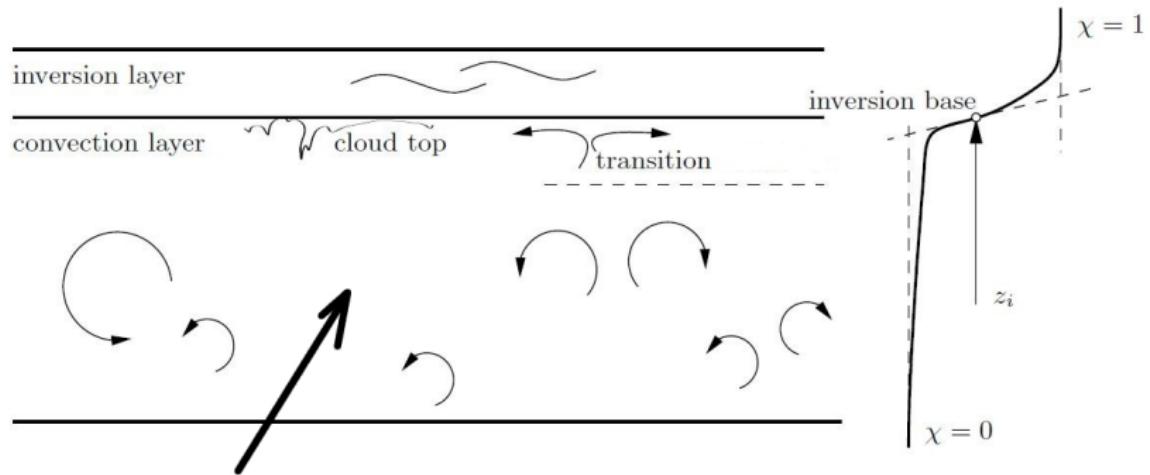
Constant thickness

$$h = \frac{1 - \chi_i}{\frac{\partial \langle \chi \rangle}{\partial z}(z_i, t)}$$



Constant entrainment rate

Convection Layer



Convection Layer

$$\frac{d}{dt} \int_{-\infty}^{z_i} \langle \chi \rangle dz = \kappa \frac{\partial \langle \chi \rangle}{\partial z}(z_i, t) - \langle w' \chi' \rangle(z_i, t) + \chi_i w_e \simeq w_e$$

Then, w_e determines also scaling inside the turbulent zone.

From functional dependence between b and χ , **reference buoyancy flux**

$$B_s = w_e |b_s| / \chi_s = (0.1 f_1 \chi_c^{2/3} / \chi_s) (\kappa b_s^4)^{1/3}$$

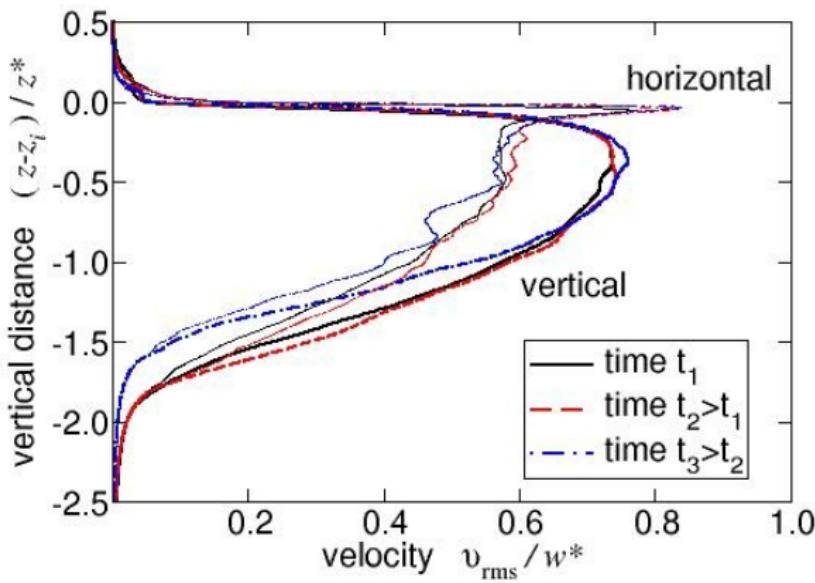
and then **convection scales**

$$z^* = (1/B_s) \int B dz \quad , \quad w^* = (z^* B_s)^{1/3}$$

characterize (some statistics of) the turbulent region (Deardorff, 1980)



Turbulent Velocity Fluctuations



- Self-preservation.
- Anisotropy.
- Inhomogeneity.



Temporal Evolution of Convection Scales

Integrating transport equation for $q^2/2$,

$$\frac{\partial q^2/2}{\partial t} = -\frac{\partial T}{\partial z} + B - \varepsilon ,$$

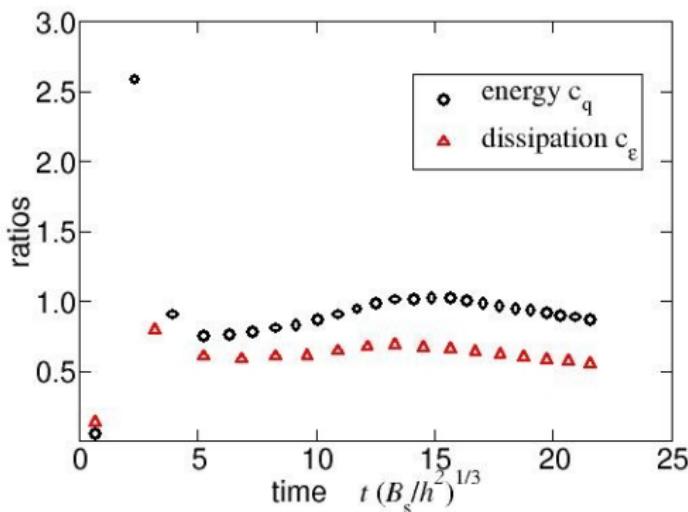


Temporal Evolution of Convection Scales

Integrating transport equation for $q^2/2$,

$$\frac{d}{dt}(c_q z^* w^{*2}) = 2(1 - c_\varepsilon) w^{*3}$$

$$c_\varepsilon = \frac{1}{w^{*3}} \int \varepsilon dz, \quad c_q = \frac{1}{w^{*2} z^*} \int q^2 dz$$



Constant $c_q(D, \chi_s)$

Constant $c_\varepsilon(D, \chi_s)$

With def. $w^{*3} = B_s z^*$, closed system for $w^*(t)$ and $z^*(t)$



Temporal Evolution of Convection Scales

Solution:

$$z^*(t) = z^*(t_1) \left[1 + (2f_2/3) \frac{t - t_1}{[z^*(t_1)^2/B_s]^{1/3}} \right]^{3/2}$$
$$w^*(t) = (1/f_2) dz^*/dt , \quad f_2 \simeq 0.5$$

Scalings:

$$z^* \propto t^{3/2} \quad w^* \propto t^{1/2} \quad b^* \propto t^{-1/2}$$

Time scale:

$$(z^*{}^2/B_s)^{1/3}$$



Stratocumulus

Turbulence does not break the cloud top, but enhance mixing up to a linear entrainment rate.

Some numbers:

$$w_e \simeq 0.16 \text{ mm/s}; h \simeq 0.1 \text{ m}; B_s \simeq 10^{-5} \text{ m}^2/\text{s}^3; z^* \simeq 2.5 \text{ m}; w^* \simeq 30 \text{ mm/s.}$$

From previous growth rates, 100 m reached in about 45 min. Then, $w^* \simeq 0.1 \text{ m/s}$; still \ll measurements of 1 m/s (radiative forcing).

Structure:

Vertical interface displacement $\delta = w^{*2}/b_1$ small and $Ri^* = z^*/\delta \propto t^{1/2}$. Internal Richardson number $Ri_{(I)} = h/\delta \propto t^{-1}$ decreases, but order 1 only after $z^* = 300 \text{ m}$.



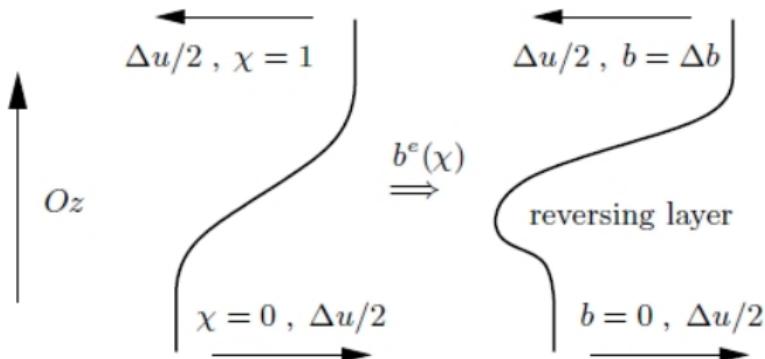
Conclusions

The cloud-top entrainment instability cannot explain break-up

- Evaporative cooling effects are one order of magnitude too small.
- Buoyancy reversal w/o mean shear depends on molecular props.

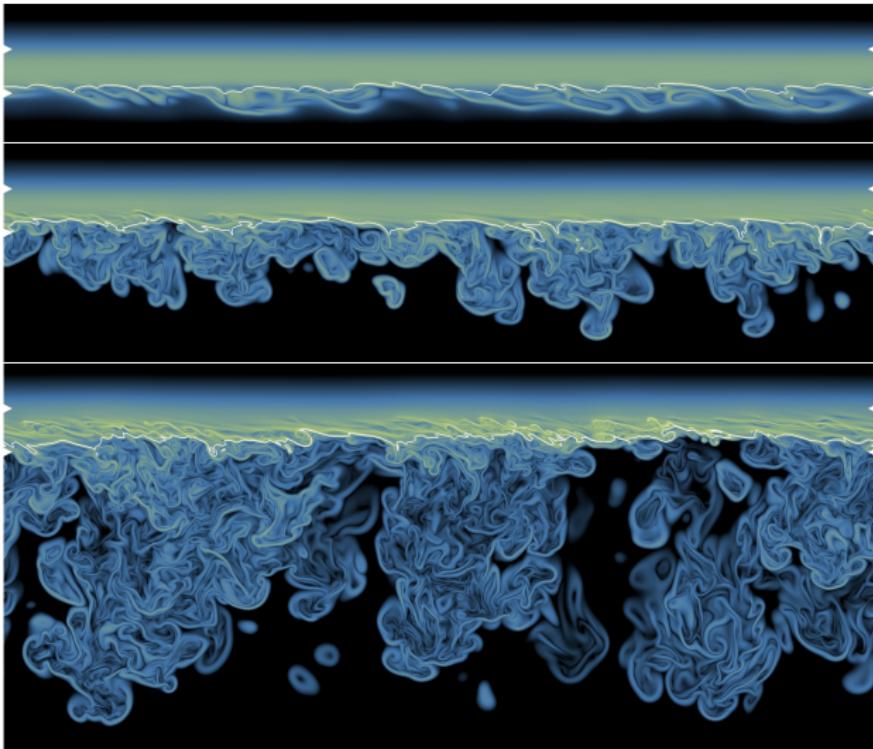


Mean Vertical Shear



$$\{\nu, \kappa, \Delta b, b_s, \chi_s, \Delta u\} \Rightarrow \{Pr, D = -b_s/\Delta b, \chi_s, (\Delta u)^3/(\nu \Delta b)\}$$

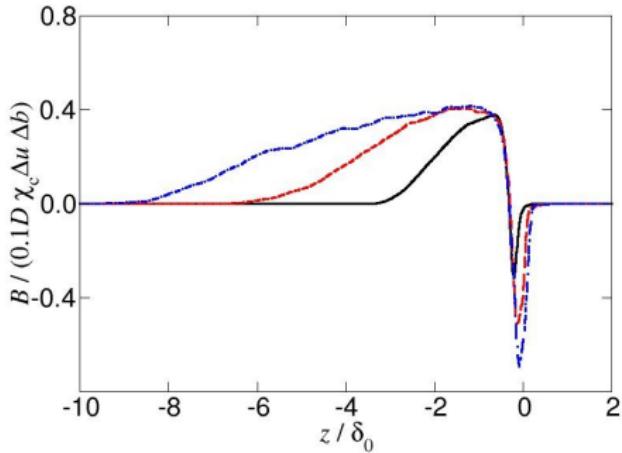
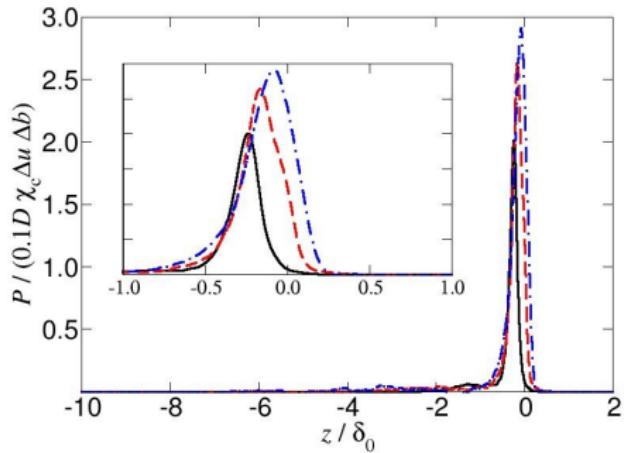
Turbulent Inversion Layer



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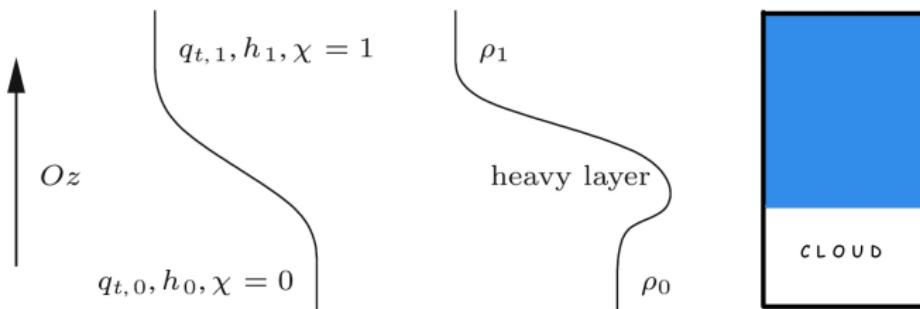
Turbulent Inversion Layer



Formulation

Two-fluid formulation ($St \simeq 0.01$, $Sv \simeq 0.3$, $\phi_d \simeq 10^{-6}$)

Mixture fraction χ (Albrecht et al. 1985; Bretherton 1987; JPM et al. 2010)

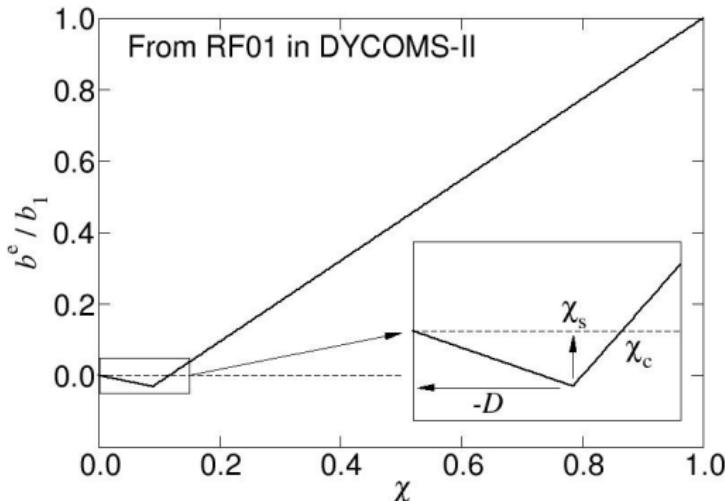


$$\frac{q_t - q_{t,0}}{q_{t,1} - q_{t,0}} - \frac{h - h_0}{h_1 - h_0} \rightarrow 0 \quad \frac{\mathbf{q}_t - \mathbf{q}_{t,0}}{\mathbf{q}_{t,1} - \mathbf{q}_{t,0}} = \frac{\mathbf{h} - \mathbf{h}_0}{\mathbf{h}_1 - \mathbf{h}_0} = \chi$$

Governing Equations

Boussinesq + Mixture Fraction χ + Non-Linear Eqn. State

$$\begin{aligned}\partial_t u_k &= -\partial_i(u_k u_i) - \partial_k p + \nu \partial_i \partial_i u_k + b \delta_{k3}, \quad \partial_i u_i = 0 \\ \partial_t \chi &= -\partial_i(\chi u_i) + \kappa \partial_i \partial_i \chi \\ b &= b^e(\chi; b_1, b_s, \chi_s)\end{aligned}$$



Parameter space:

$$\{\nu, \kappa, b_1, b_s, \chi_s\}$$



$$\{Pr = 1, D = -b_s/b_1, \chi_s\}$$

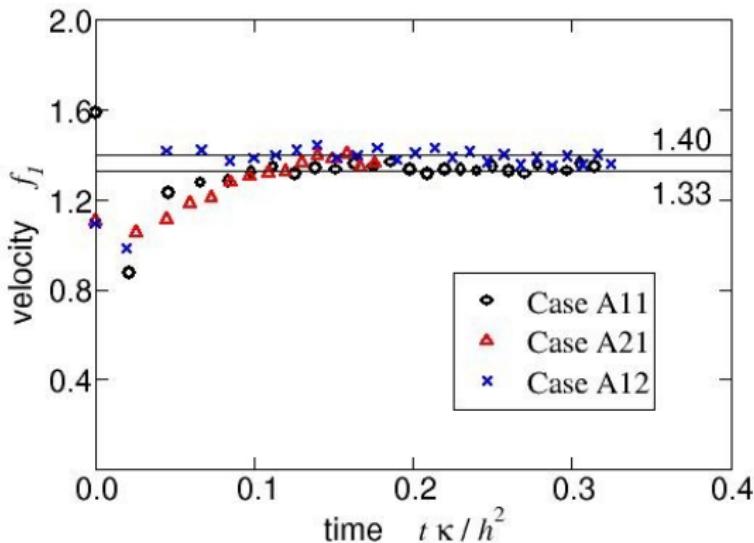
Mean Entrainment Rate

Marginally stable thermal boundary layer $\chi_c h$:

$$\frac{(\chi_c h)/(\kappa/h)}{\nu/(\chi_c h |b_s|)} \simeq 10^3 \quad \Rightarrow \quad h \simeq (10/\chi_c^{2/3})(\kappa^2/|b_s|)^{1/3}$$

$$f_1 = \frac{w_e}{(\chi_c^{2/3}/10)(\kappa|b_s|)^{1/3}}$$

$$f_1 = f_1(D, \chi_s)$$



Previous Work

Siems et al. (1990), Shy and Breidenthal (1990), Siems et al. (1992)

- Tank experiments with liquid mixtures. Mechanically driven ICs.
- Definition of the problem in terms of $D = -b_s/b_1$ and χ_s .
- Sims. Almost laminar behavior for $D \simeq 0.04$ (real conditions).
- Small reversal ($D \ll 1$) cannot explain cloud break-up.

Wunsch (2003)

- Stochastic models.
- Confirms previous results.
- Points to possible relevance of diffusion at cloud interface.

What is really going on at the interface?



Further Discussion

	z^*/h	$\eta/\Delta x$	z^*/λ_z	λ_z/η	u'/w^*	w'/w^*	Re_t	Re_λ	Re^*	Ri^*	Ra^*
A11	24	1.2	19	28	0.84	0.74	1800	220	4800	590	0.4×10^9
A21	39	0.9	26	31	0.86	0.78	2400	250	8000	293	1.1×10^9
A12	39	1.2	19	28	0.90	0.76	1600	200	4800	716	0.5×10^9

TABLE 2. Length-scale ratios, turbulence intensities and derived quantities at the final time t_2 . Reynolds numbers $Re_t = (g^2/2)^{1/2}/(\varepsilon v)$, $Re_\lambda = w' \lambda_z / v$ and $Re^* = z^* w^* / v$; convection Richardson number $Ri^* = b_1 z^* / w^{*2}$; Rayleigh number $Ra^* = z^{*3} |b_s| / (\kappa v)$; Nusselt number $Nu^* = w_e z^* / \kappa = z^* / h$. Maximum values are used for the mean turbulent dissipation rate ε and the turbulence intensities.

- Unsteady free convection; $Nu^*(t)/(Ra^*(t))^{1/3} = 0.1 f_1 \chi_c^{2/3}$ const.
- Turbulent mixing across a density interface; $Ri^*(t)$ increasing.
- Stratocumulus.

