Turbulence Statistics along Gradient Trajectories

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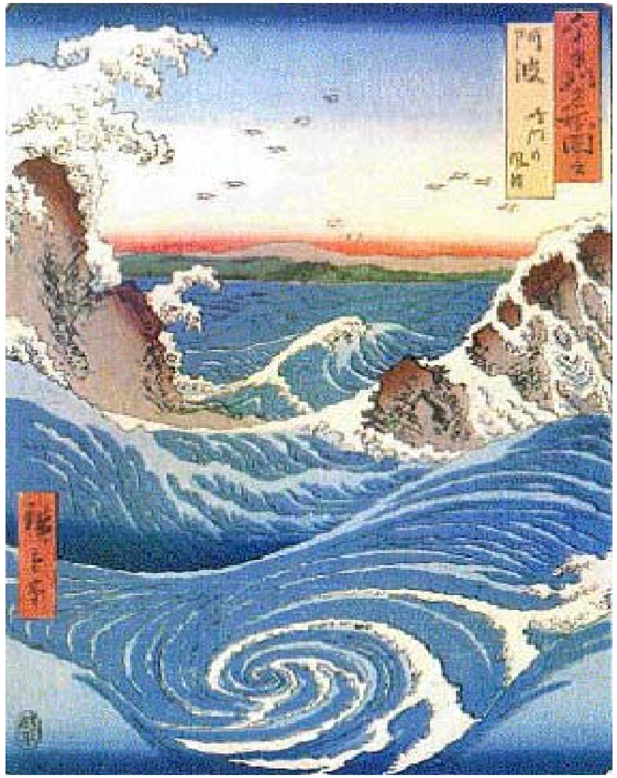
RWTH AACHEN UNIVERSITY

Contents

Introduction

Turbulence: phenomenologically a fluid regime characterized by chaotic and stochastic property changes.

by Leonardo da Vinci



Various applications

combustors aircrafts meteorology

Methods used in turbulence research

- Experiments
  - advantages: real physical phenomena; high Reynolds numbers
  - disadvantages: only limited access to full 3D structures, often straight line measurements
- Direct numerical simulation (DNS)
  - advantages: high spatial resolution, entire velocity and pressure fields
  - disadvantages: artificial effects from boundary conditions, low Reynolds numbers.
- Stochastic theory and scaling

Kolmogorov's (1941) first hypothesis of similarity

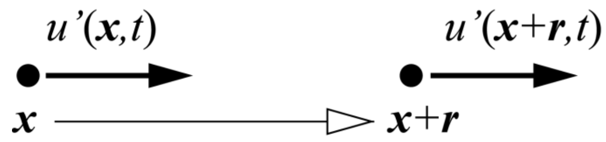
For locally isotropic turbulence

the n-point distribution functions F_n

are uniquely determined by the viscosity ν and the dissipation ε

Inertial range: $\nu \to 0$, $\varepsilon -$ **scaling** !

Two point statistics along a straight line



structure function of moment *m*: $B_m = \left\langle \left(u'(\boldsymbol{x}+\boldsymbol{r},t) - u'(\boldsymbol{x},t) \right)^m \right\rangle$

Kolmogorov's equation

$$\underbrace{3\frac{\partial B_2}{\partial t}}_{= \text{ 0 for steady state}} + \frac{1}{r^4}\frac{\partial}{\partial r}(r^4 B_3) = -4\varepsilon + \underbrace{\frac{6\nu}{r^4}\frac{\partial}{\partial r}\left(r^4 \frac{\partial B_2}{\partial r}\right)}_{= 0 \text{ for } \nu \to 0}$$

exact result: $B_3 = -\frac{4}{5}\varepsilon\, r$

scale invariance assumption: $B_m \propto (\varepsilon\, r)^{\zeta_m}$

with $\zeta_m = \frac{m}{3}$ for all moments

If Kolmogorov’s scale invariance ***was*** exact, the task of computing practical flows would be relatively simple.

The eddy viscosity relating the third to the second structure function would then by

$$\nu_t = \alpha \cdot r\sqrt{B_2(r)}$$

where α is a universal constant (Oberlack & Peters, 1993).

Scale invariance would then provide a general framework for developing closure models.

Unfortunately, scale invariance is *not* exact

Examples:

- Anomalous scaling - scaling exponents ζ_m depart from $m/3$ for $m \neq 3$
- Derivative skewness and flatness are Reynolds number dependent

K. R. Sreenivasan, R. A. Antonia, Annu. Rev. Fluid Mech., 1997

Why should we care?

We have two-equations models of turbulence to close the Reynolds averaged Navier Stokes equations (RANS),

even better: we have Large Eddy Simulations

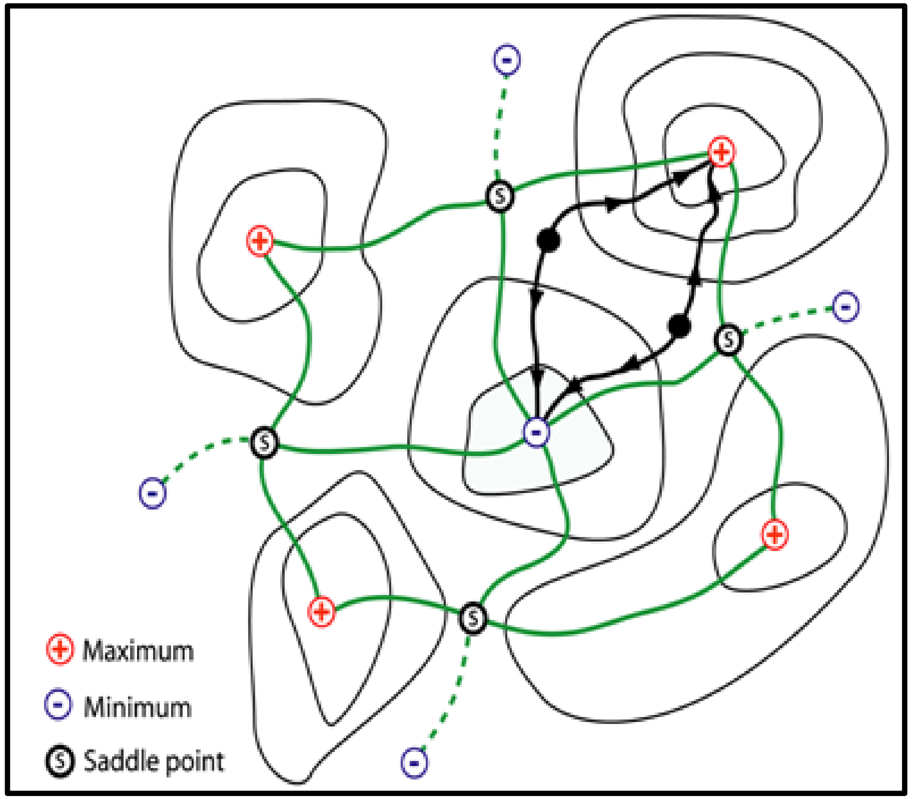
But:

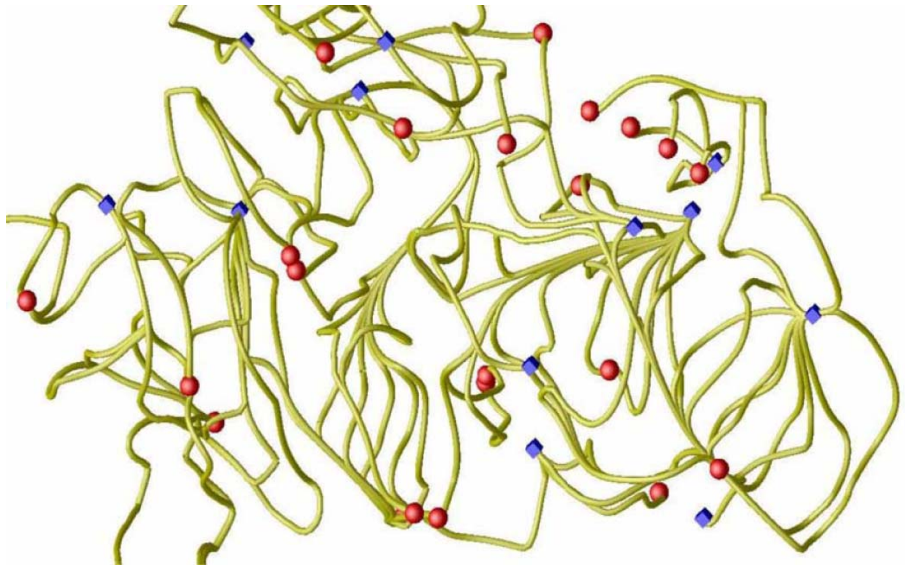
The basic argument in favor of modeling unclosed expressions is

scale invariance for the unresolved scales

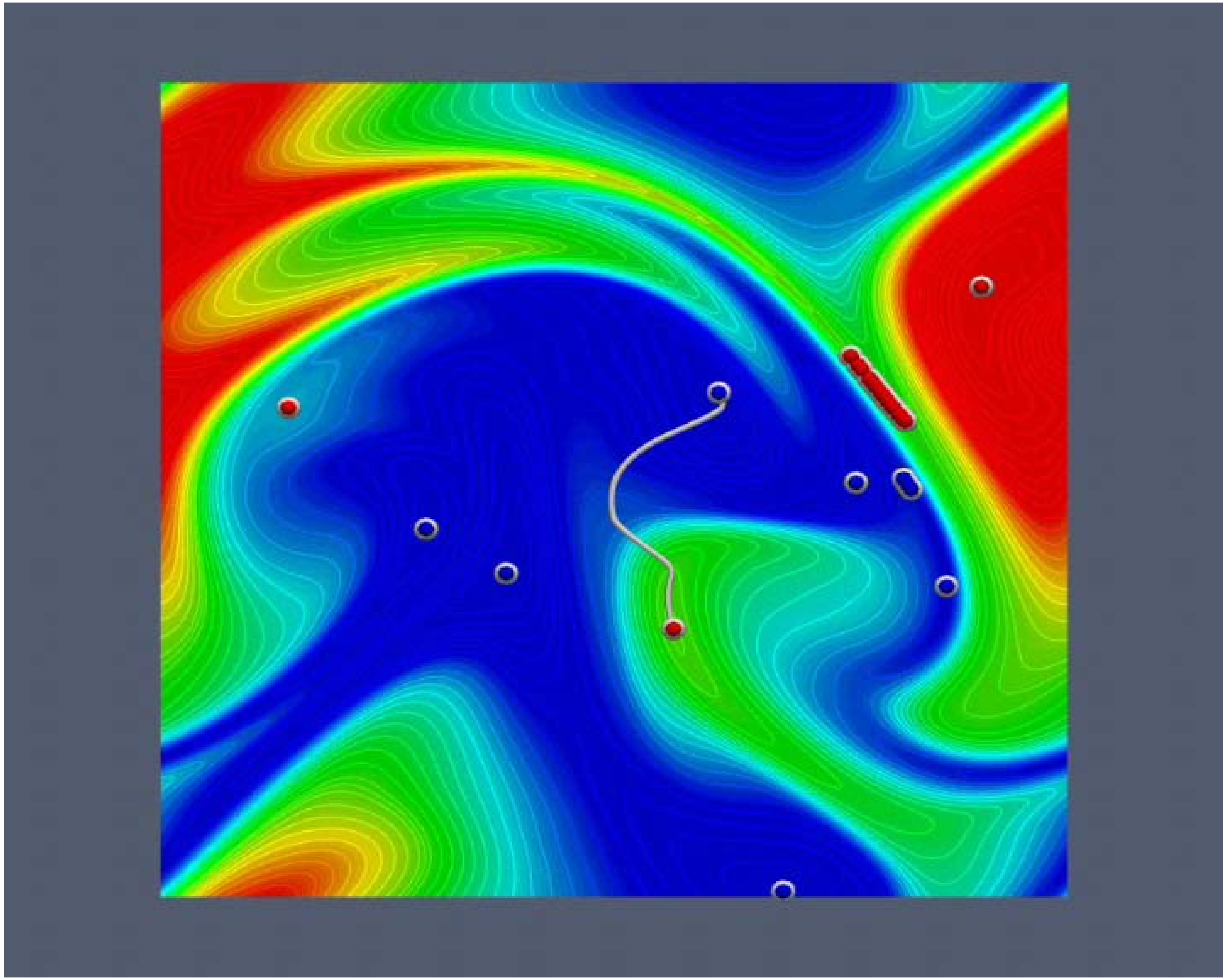
Conditional scaling along gradient trajectories in a scalar field

A scalar field can be that of a passive scalar, the instantaneous kinetic energy or the instantaneous dissipation

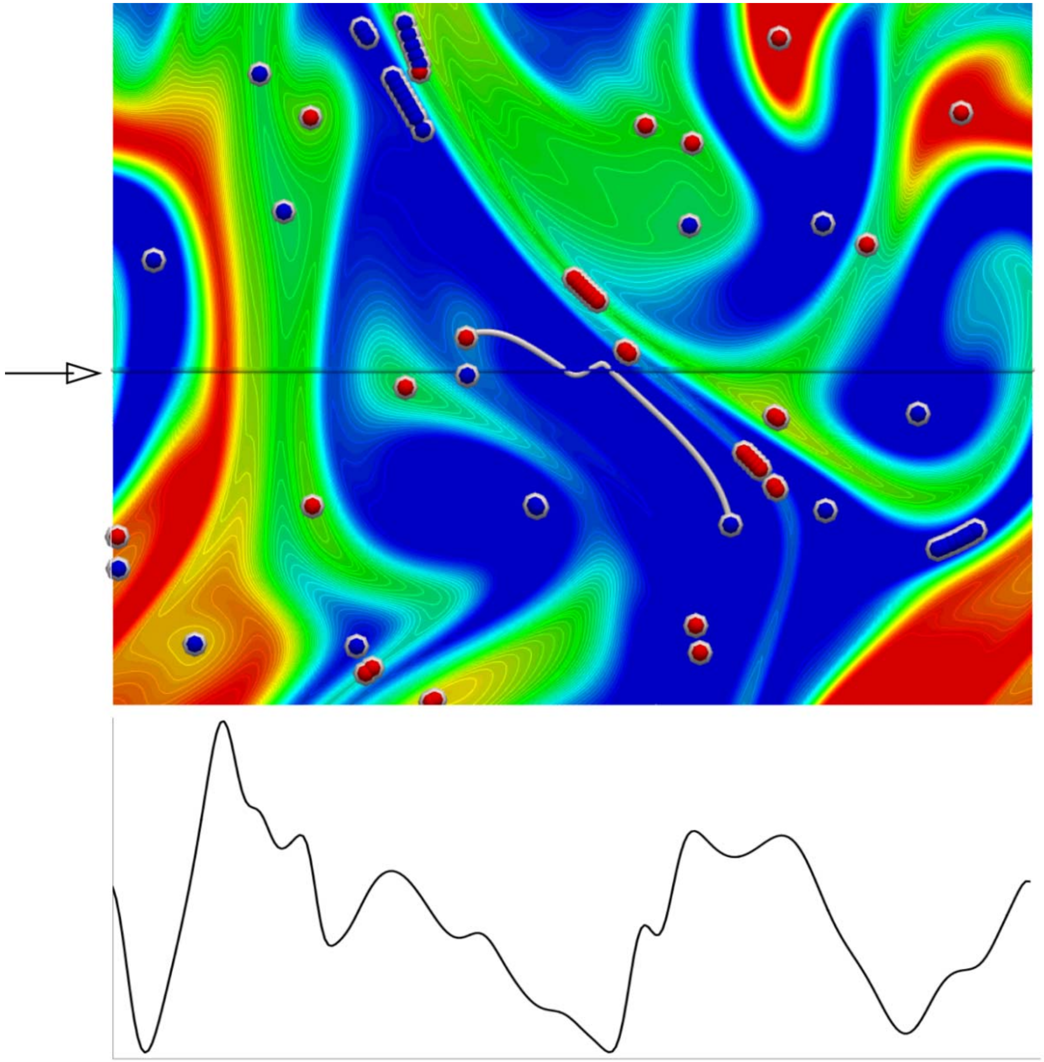




Chaotic motion of gradient trajectories in a 2-D scalar field

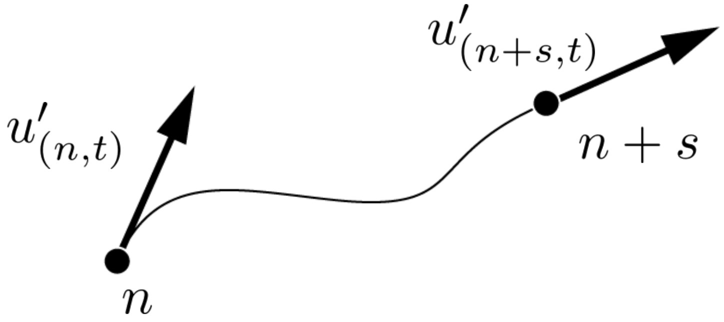


Cliff-ramp structure in the scalar field



(distribution along a horizontal line)

Two point statistics along gradient trajectories



Scalar gradient: $$\phi'_{,n} = \frac{\partial \phi'}{\partial n}$$

Two point correlation:

$$C_2 = \langle \phi'_{,n_{(n+s,t)}} \phi'_{,n_{(n,t)}} \rangle$$

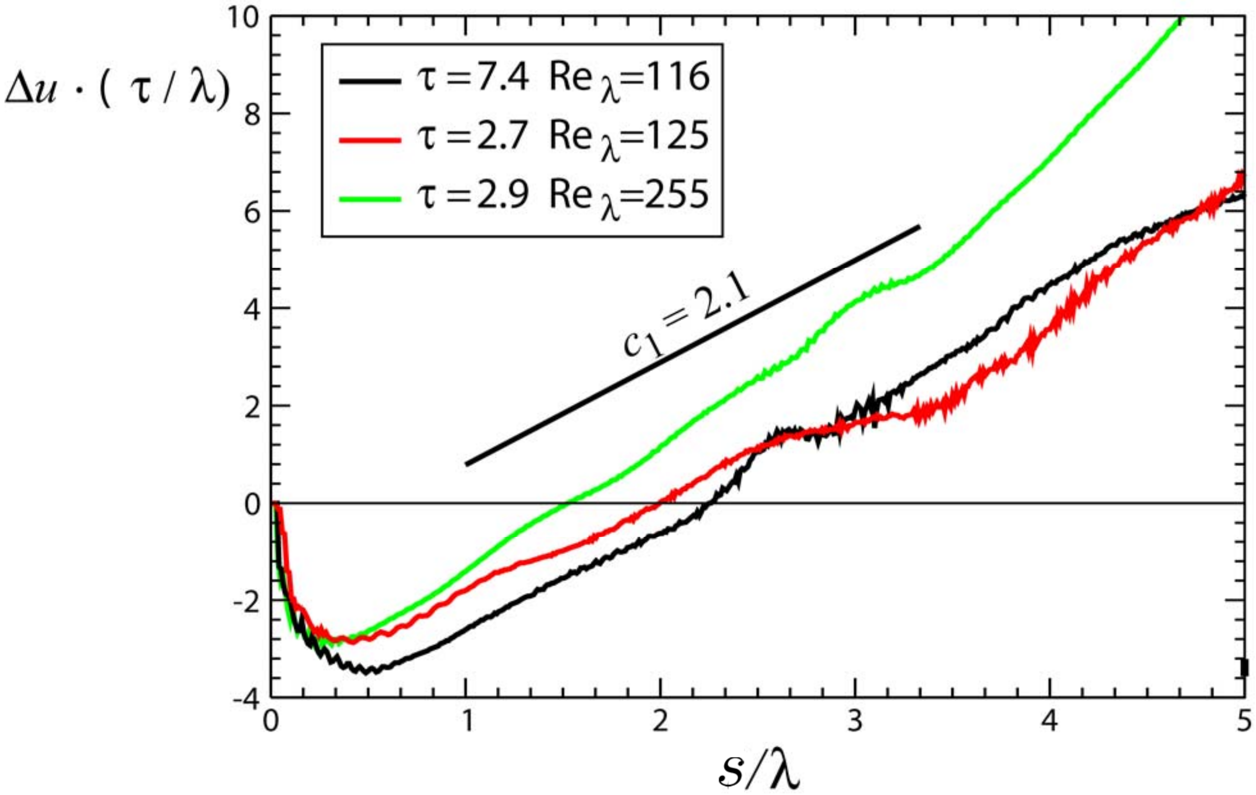
$$\underbrace{\frac{\partial C_2}{\partial t}}_{\propto\ -C_2/\tau \text{ for } t \to \infty} + \underbrace{\frac{\partial}{\partial s} \langle (u'_{(n+s,t)} - u'_{(n,s)}) \phi'_{,n_{(n+s,t)}} \phi'_{,n_{(n,t)}} \rangle}_{\frac{\langle u'_{(n+s,t)} - u'_{(n,s)} \rangle C_2}{s}} = \underbrace{D(...)}_{= 0 \text{ for } \nu \to 0}$$

Due to decorrelation

Result: $$\langle u'_{(n+s,t)} - u'_{(n,s)} \rangle = c_1 \frac{s}{\tau} \text{ for } s > \lambda$$

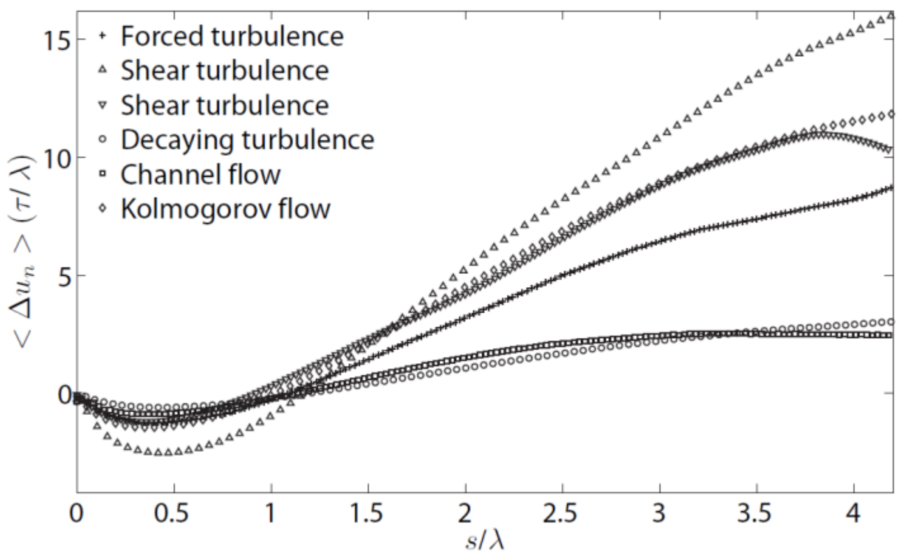
scaling parameter is the **integral time τ** rather than ε .

Normalized velocity increments along gradient trajectories in the passive scalar field in shear flow turbulence



Linear scaling $\Delta u = c_1 \frac{s}{\tau}$

Normalized velocity increments along gradient trajectories in the kinetic energy field for different flow configurations



flow type	grid	Re_λ
Homogeneous shear turbulence	2048^3	295
Homogeneous shear turbulence	1024^3	139
Isotropic homogeneous forced turbulence	1024^3	126
Kolmogorov flow	1024^3	188
Isotropic homogeneous decaying turbulence	1024^3	71

Linear scaling: $\Delta u \sim \frac{s}{\tau}$

but with different slopes

flow type	grid	Re_τ
Channel flow	512 x 512 x 385	590

Two different scalings

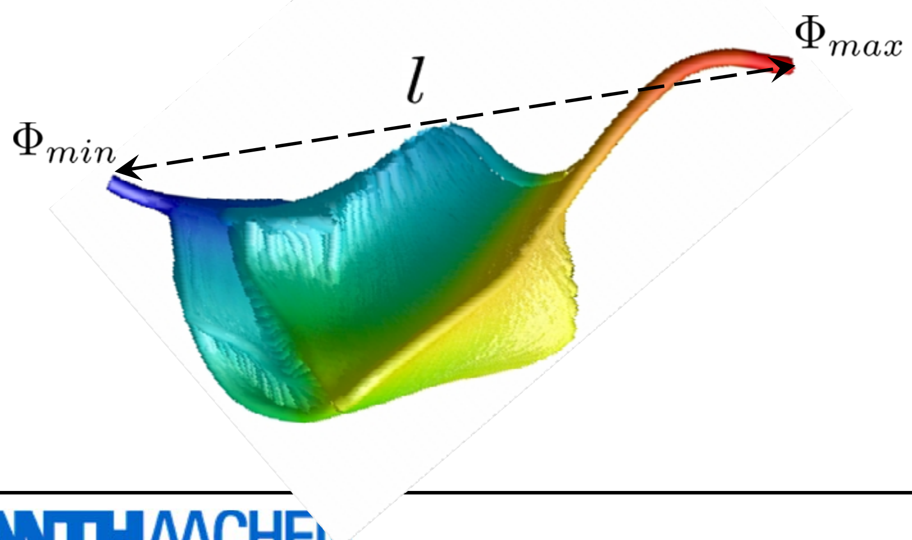
	ε-scaling	τ-scaling
velocity increment at large scales	$\delta_r u \propto (\varepsilon l)^{1/3}$	$\delta_r u \propto l/\tau$
velocity decay at small scales	$\delta_r u \propto \nu/l$	$\delta_r u \propto \nu/l$
transition at equal $\delta_r u$	$(\varepsilon l_c)^3 = \nu/l_c$	$l_c/\tau = \nu/l_c$
critical cut-off scale	$l_c = \eta = \left(\frac{\nu^3}{\varepsilon}\right)^{1/4}$	$l_c = (\nu\tau)^{1/2} = \lambda$
	Kolmogorov scale	**Taylor scale**

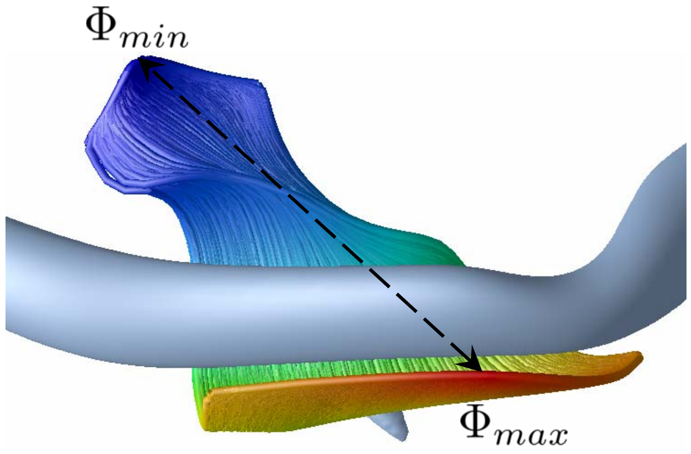
Dissipation elements

Local minimum and maximum points in the mixture fraction fluctuation field are determined by **gradient trajectories** starting from each grid cell in the directions of ascending and descending scalar gradients

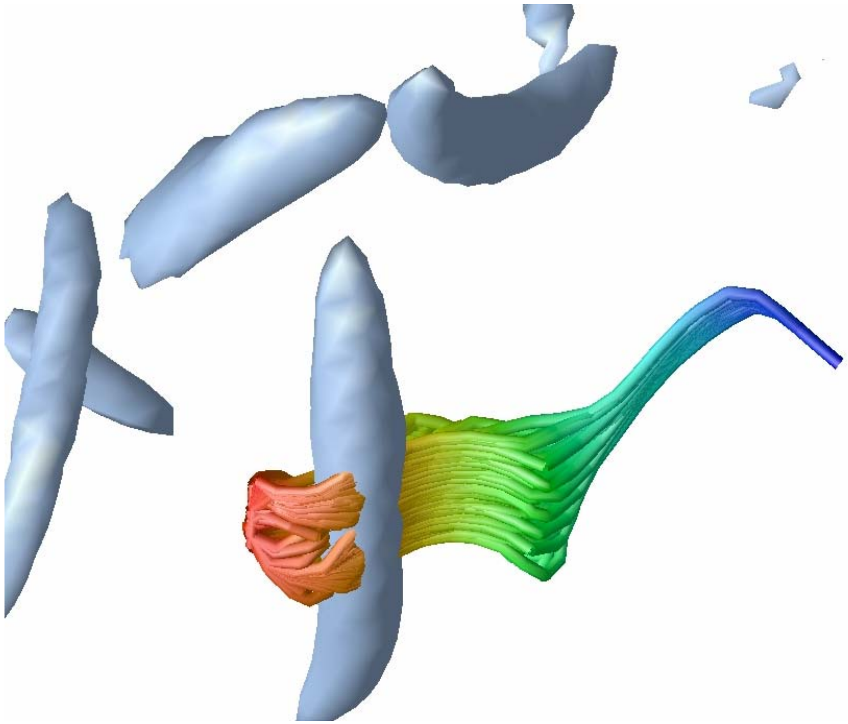
Definition:

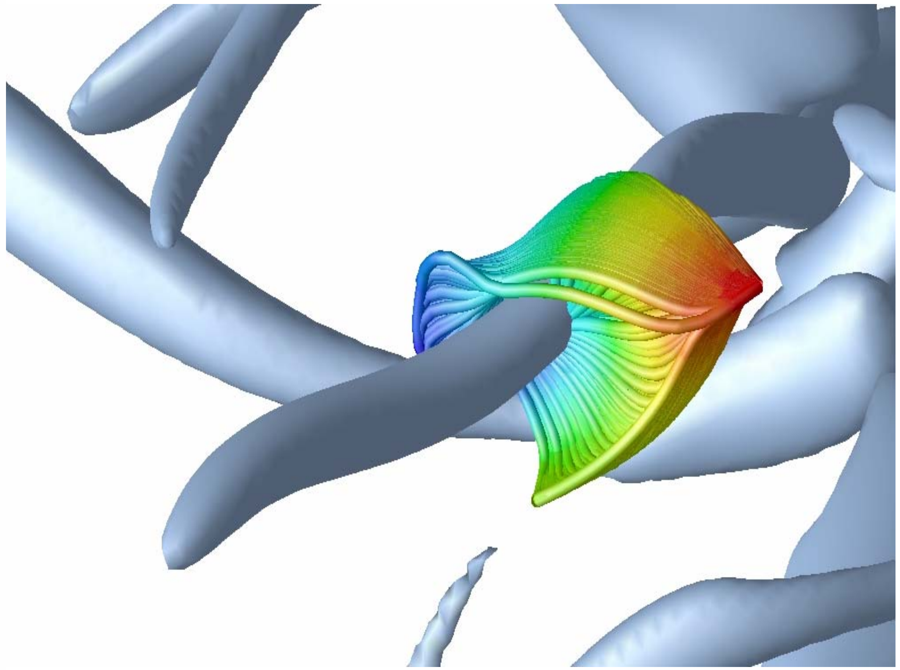
The ensemble of grid cells from which the same pair of extremal points is reached determines a spatial region defined as "dissipation element".





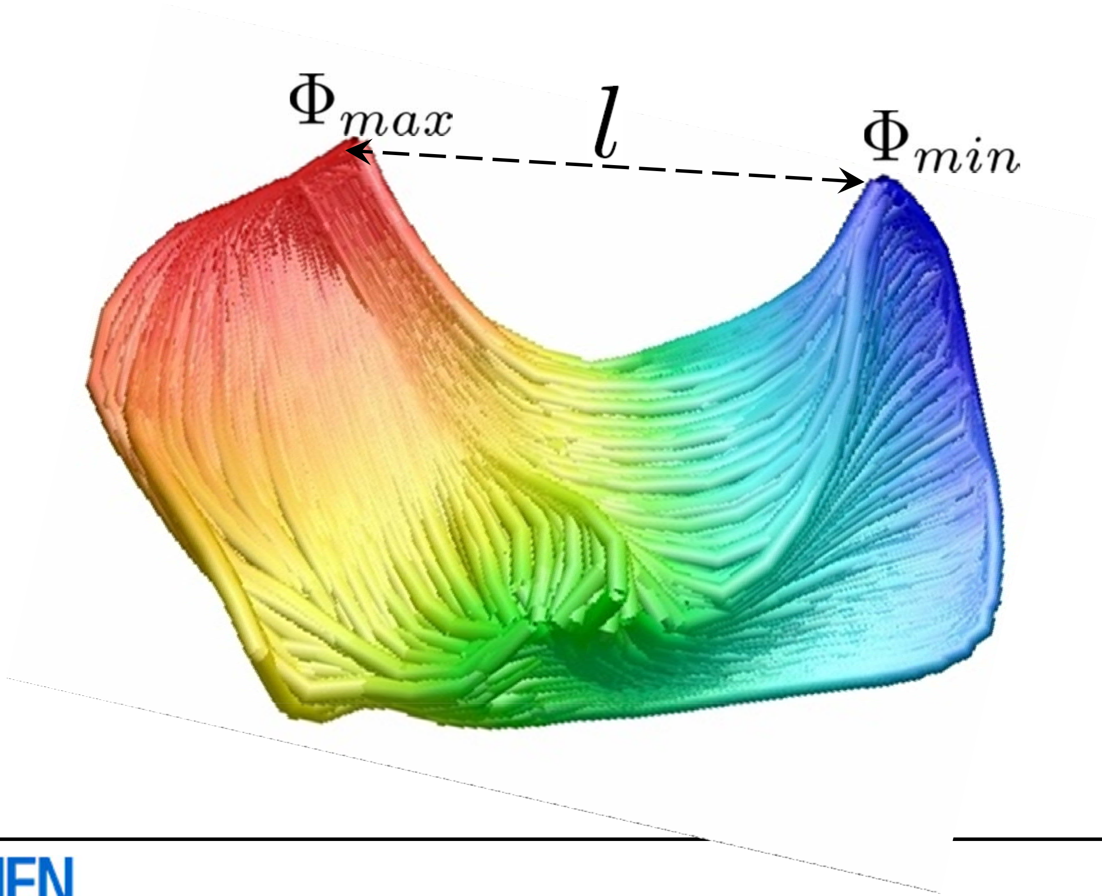
Interaction of dissipation elements with vortex tubes



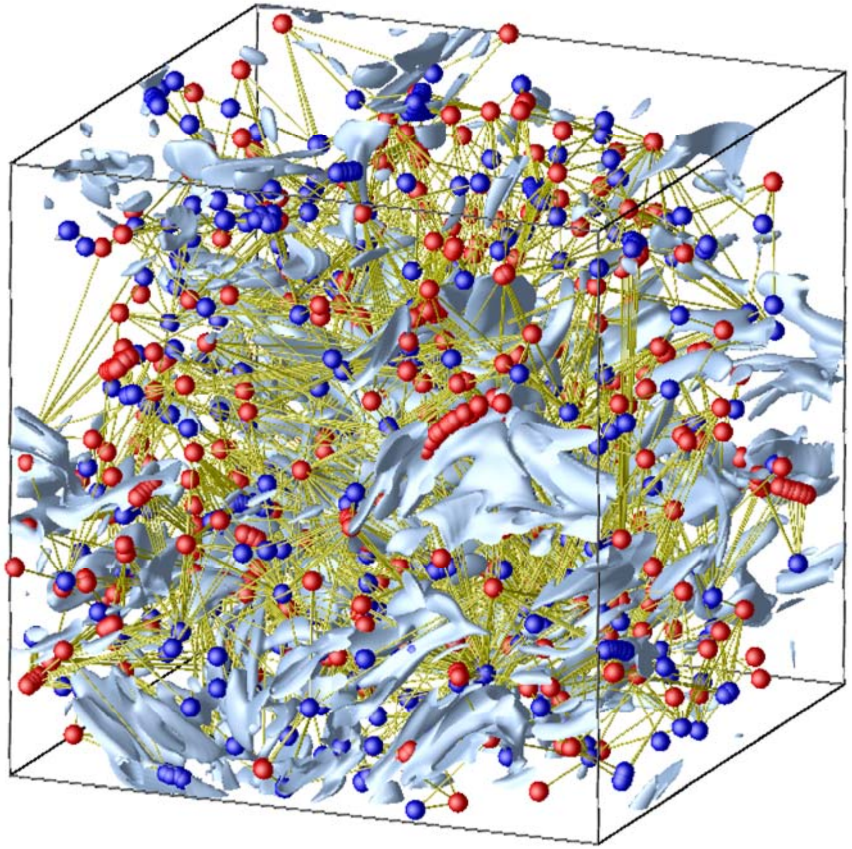


Parametric description

Among the many parameters to describe the statistical properties of dissipation elements, we have chosen l and $\Delta\phi$', which are defined as the straight line connecting the two extremal points and the scalar difference at these points, respectively.



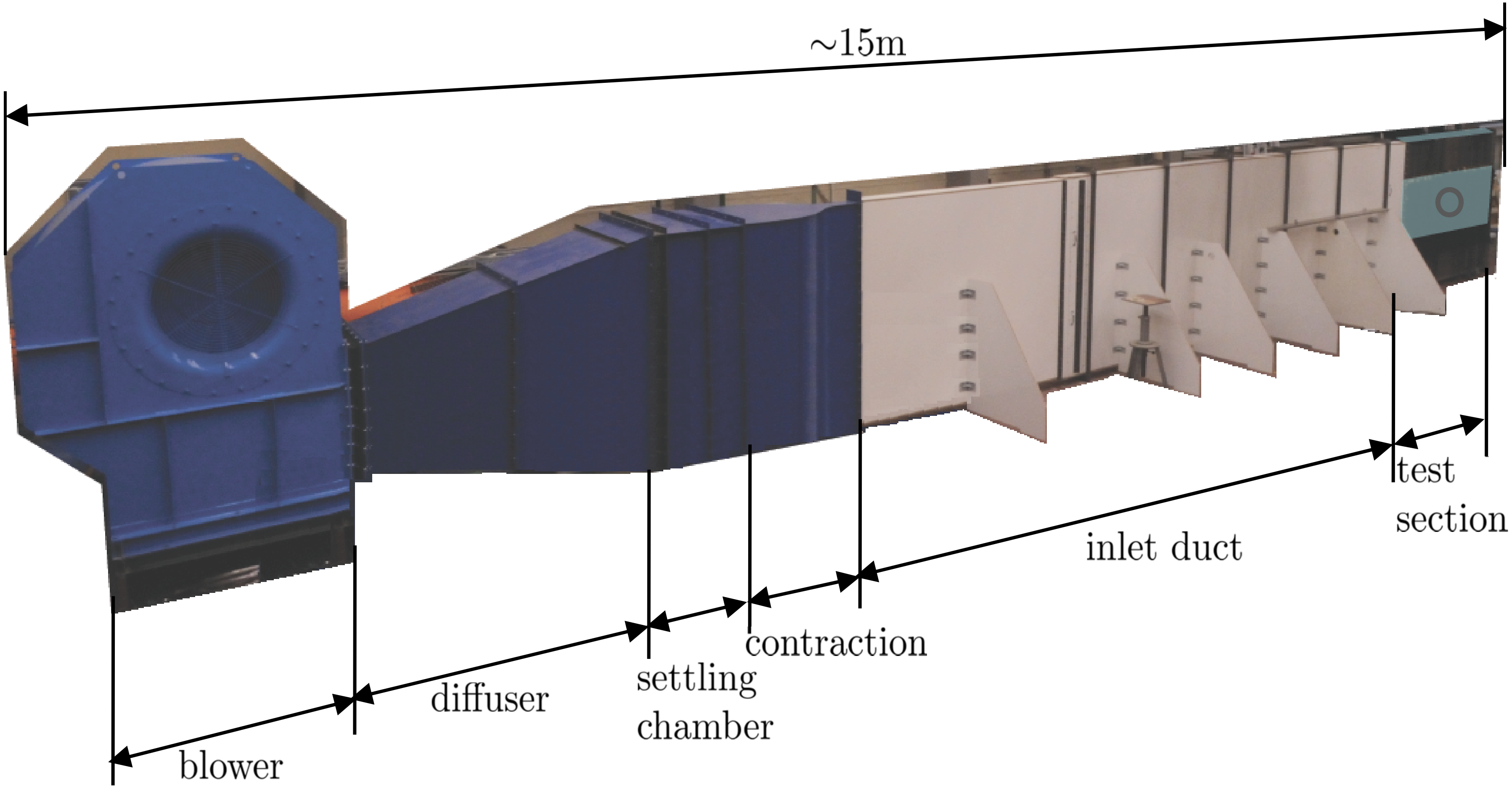
Extremal points and strain rates for the scalar field in homogeneous shear flow



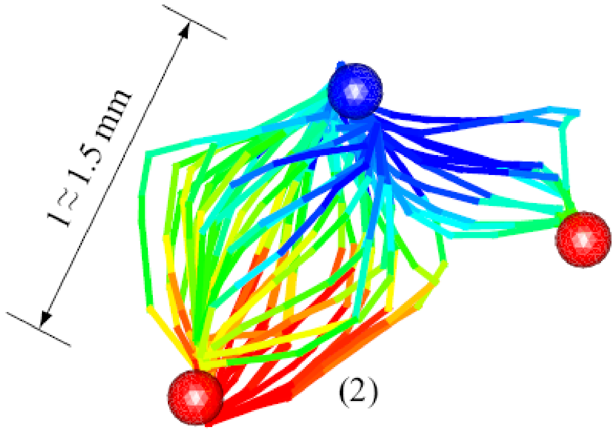
Clustering of extremal points becomes more evident.

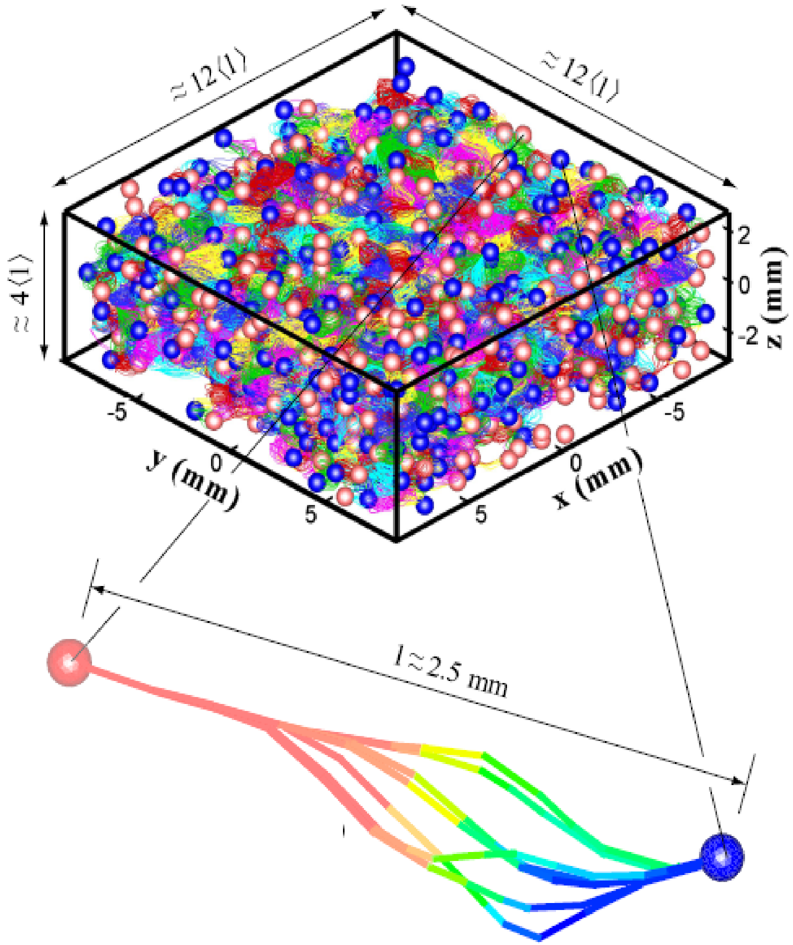
(L. Wang and N. Peters, JFM **554** (2006) 457-475)

Experimental setup in the wind tunnel of the Aerodynamics Institute at the RWTH Aachen (Prof. Schröder)



Tomographic PIV and visualisation of dissipation elements





(L.Schäfer, Physics of Fluids **23** (2011), 035106)

Experimental investigation of the mass fraction field of propane discharging from a round jet into ambient air (I)

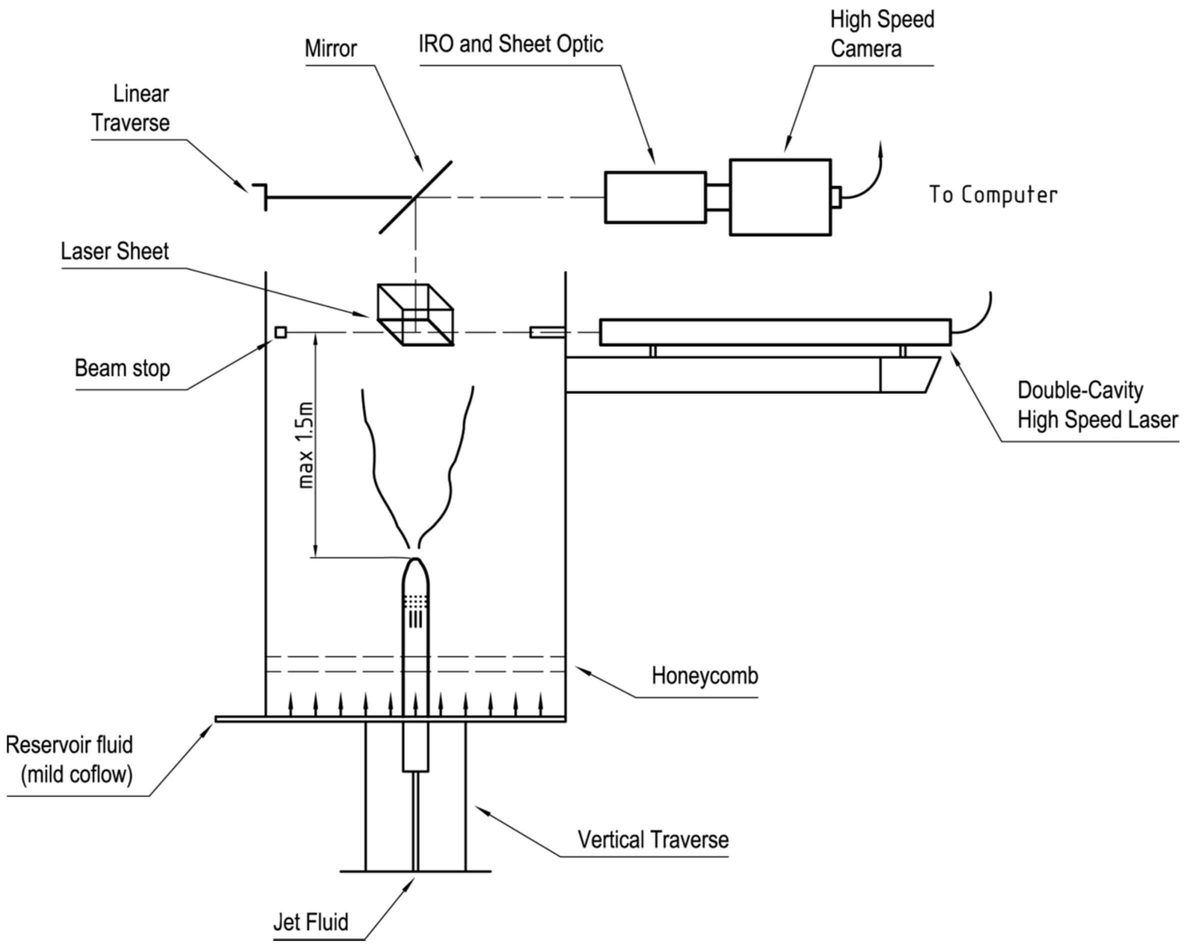
Nd:YLF* Laser

- diode pumped double cavity

Optics

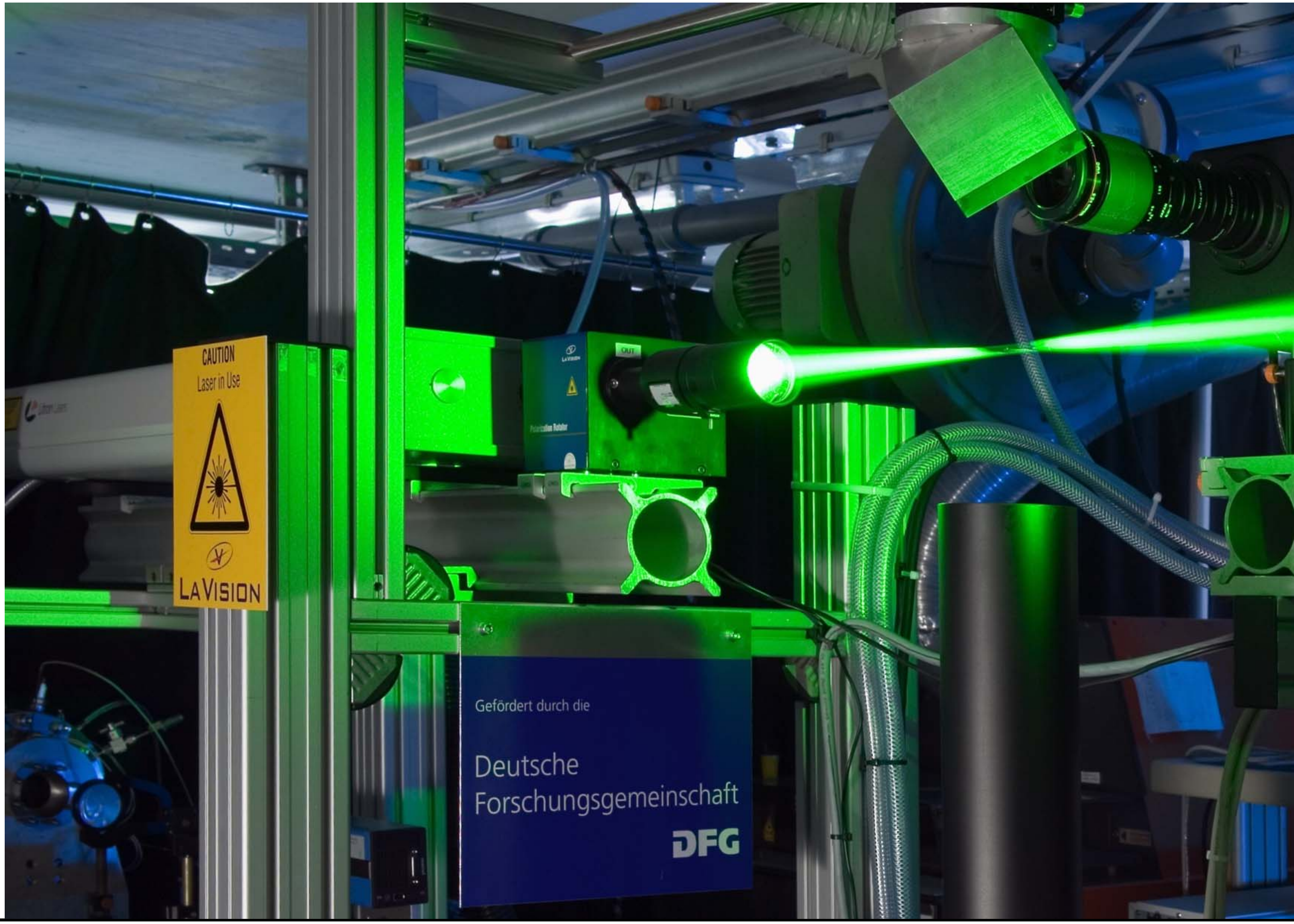
- Cam: up to 10kHz and 1024^2 pixel

Combination of Taylor's hypothesis and high-speed Rayleigh scattering

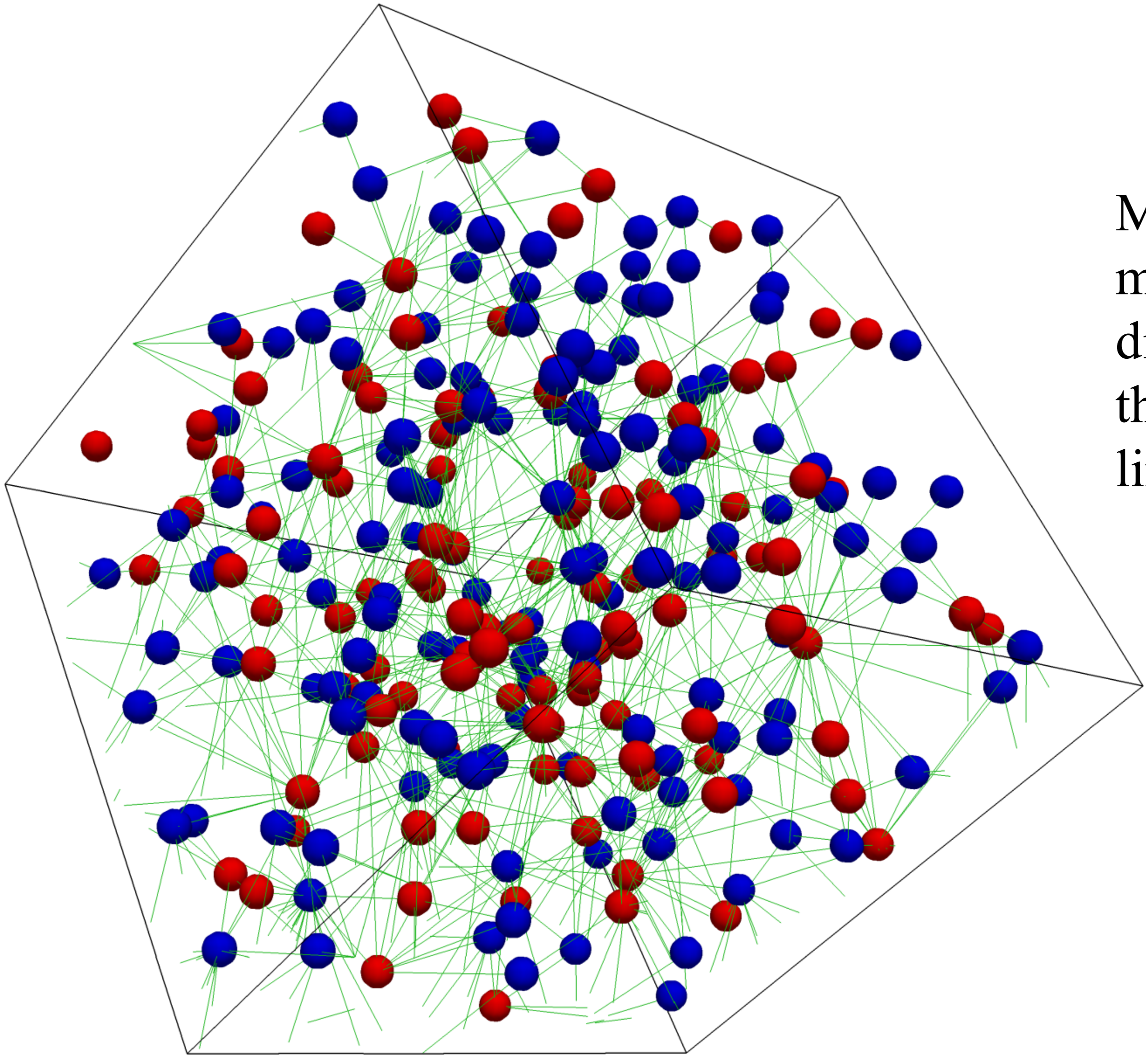


*neodymium-doped yttrium lithium fluoride

Experimental investigation of the mass fraction field of propane discharging from a round jet into ambient air (II)

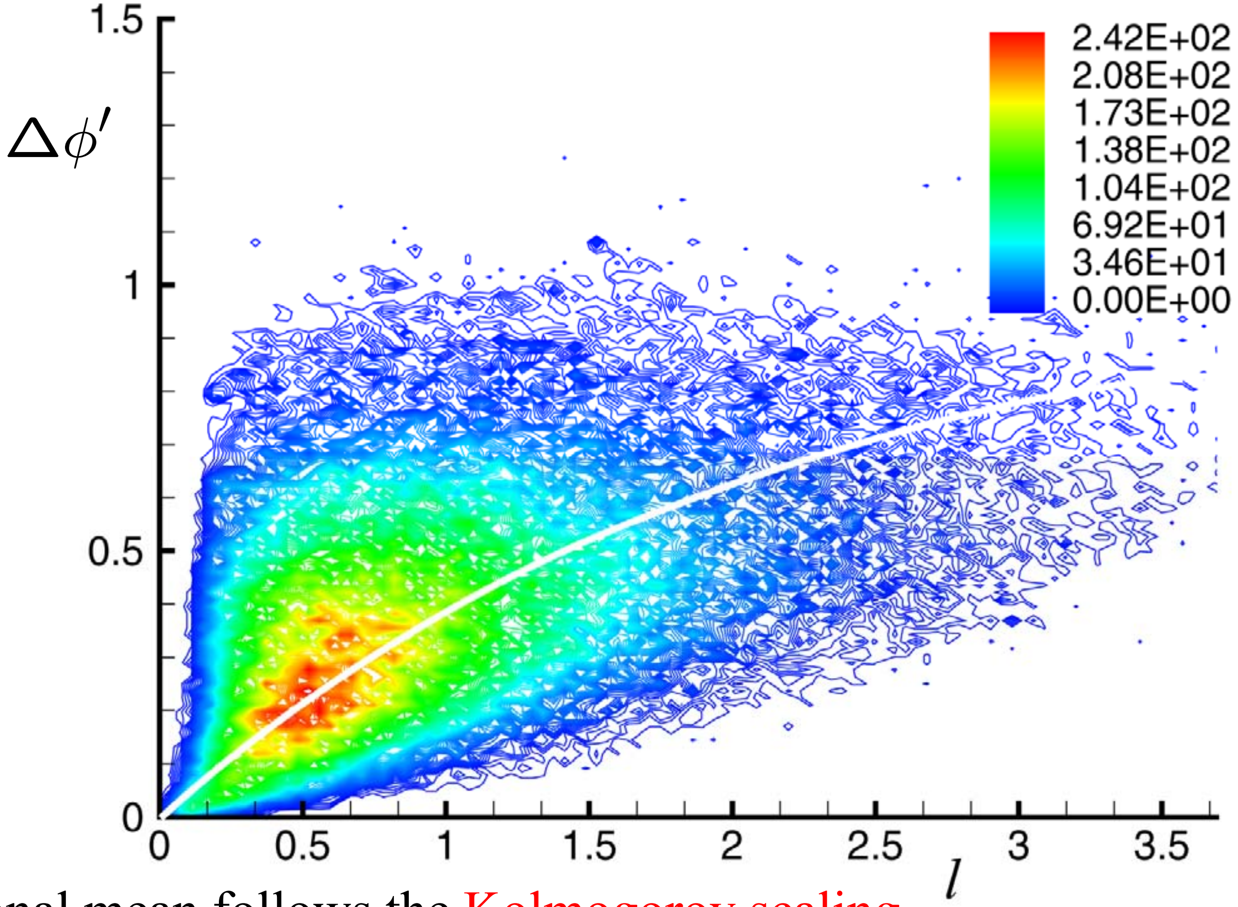


Experimental investigation of the mass fraction field of propane discharging from a round jet into ambient air (III)



Maximum (red) and minimum (blue) of dissipation elements and their linear connecting line (green)

Joint pdf of scalar difference at the extremal points and the linear length from DNS calculations

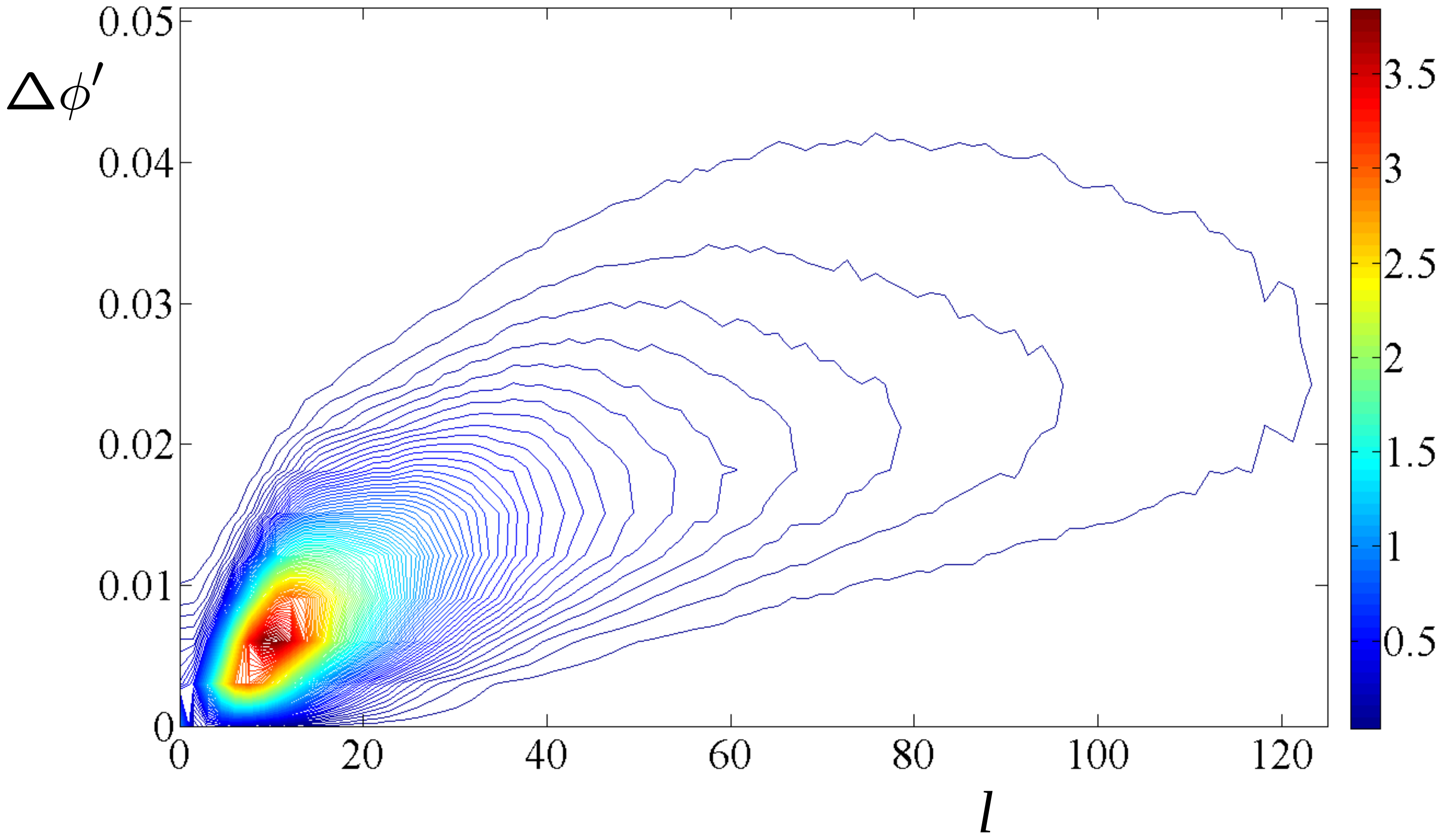


$\langle \Delta\phi' | l \rangle \sim l^{1/3}$

Conditional mean follows the Kolmogorov scaling.

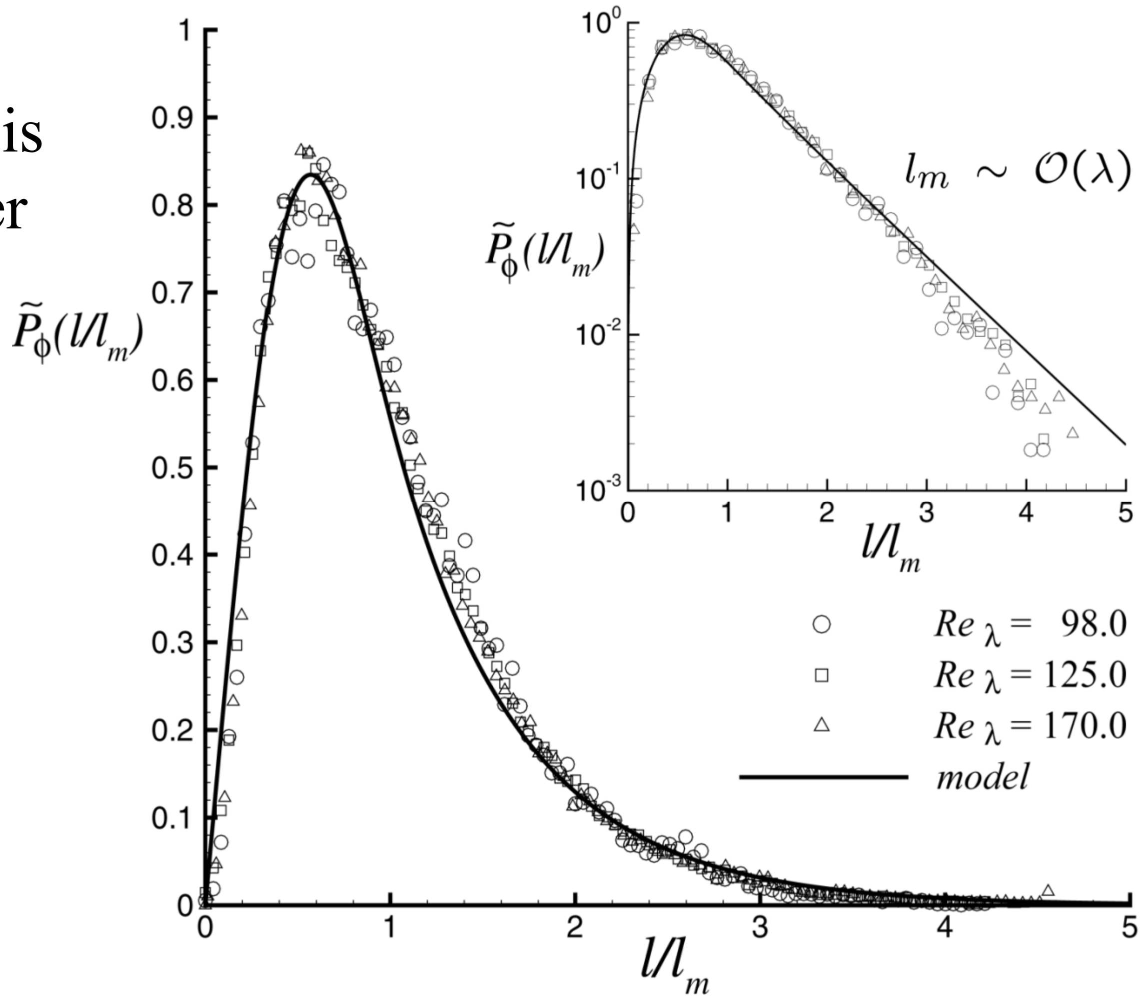
(L. Wang and N. Peters, JFM **554** (2006) 457-475)

Experimental data using Rayleigh scattering for the joint pdf of element length and scalar difference



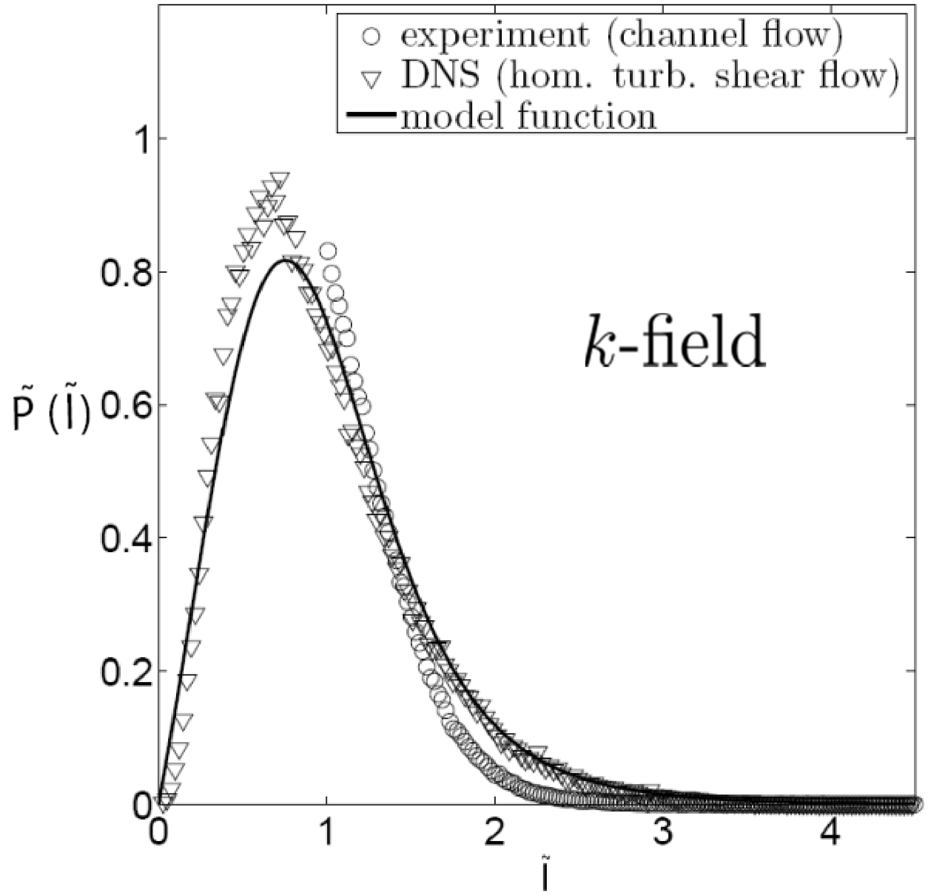
The marginal pdf of length for the passive scalar field from DNS

- The normalized shape of the pdf is Reynolds number independent



Experimental and DNS data for marginal pdf’s in turbulent channel flow

- Experimental data are from 50 test volumes
- Good agreement of pdf’s regarding exponential decay
- Average dissipation element length scales with Taylor length



(L.Schäfer, Physics of Fluids **23** (2011), 035106)

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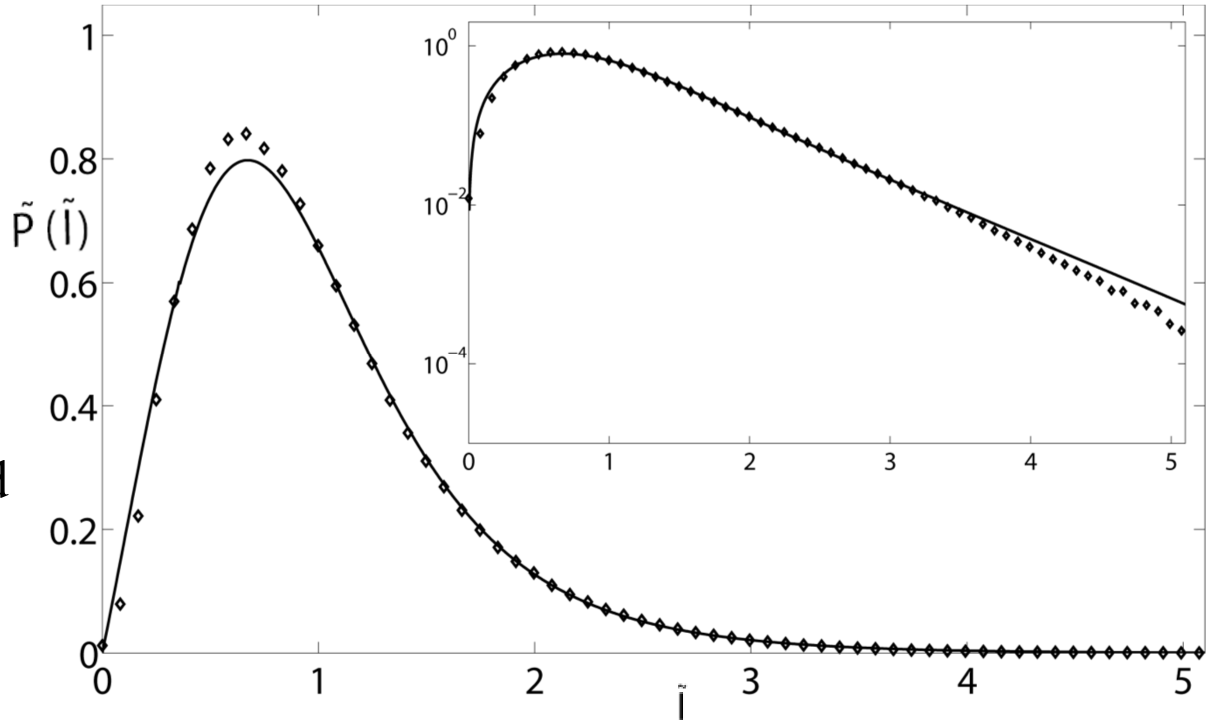
Experimental and DNS data for the scalar field

- Excellent agreement between experimental data and model solution for marginal pdf at x/d=30

- Linear increase at the origin due to diffusion

- Exponential tail modeled by Poisson process

- Very good agreement of maximum position and value



A model for the length pdf

Rapid (jump) processes:

1. The Poisson processes of random splitting and (re-) attachment.

   This gives an exponential distribution for large elements.

Slow processes

2. Continuous change of length by diffusion and straining of end points.

   Diffusive drift to origin enforces the $P(l=0)=0$.

Evolution equation for the linear length

There are four terms describing the changes of the pdf

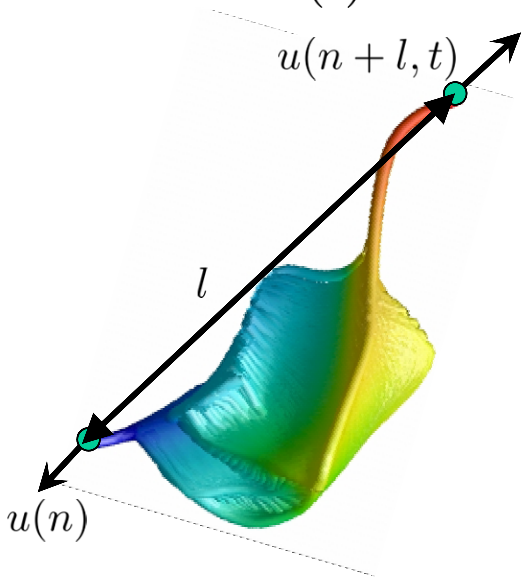
- **Generation (of small elements) by splitting**
- **Removal (of all elements) by attachment**
- **Generation and Removal (of different size elements) by strain**
- **Removal (of small elements) by diffusional drift**

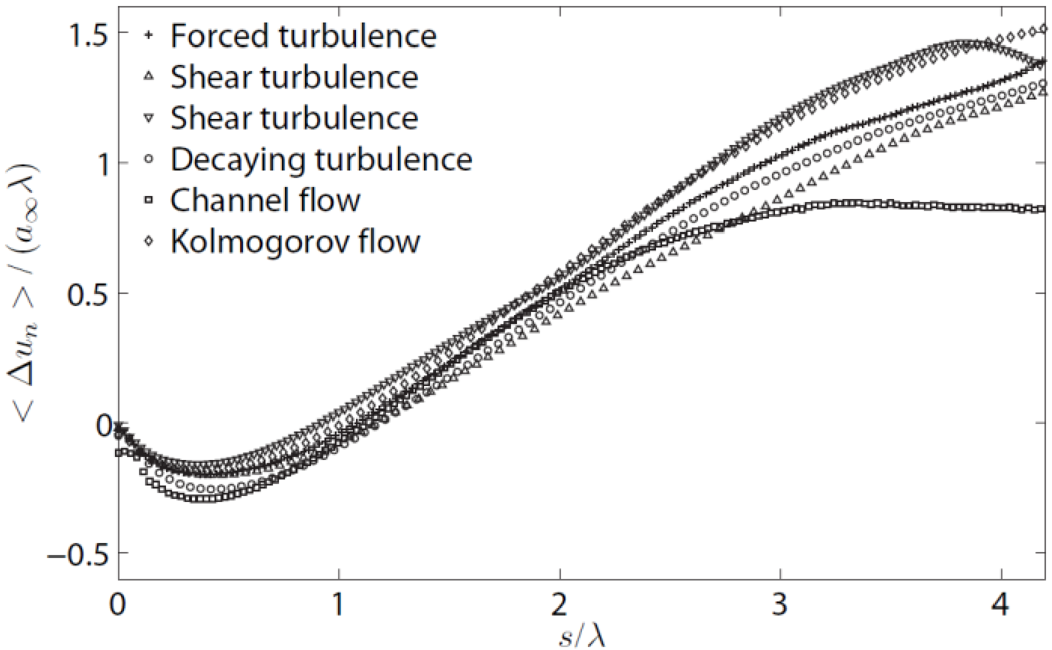
$$\frac{\partial P(l,t)}{\partial t} + \underbrace{\frac{\partial [D/l P(l,t)]}{\partial l}}_{\text{diffusional drift}} + \underbrace{\frac{\partial [a(l) l P(l,t)]}{\partial l}}_{\text{drift due to strain}} = \underbrace{\lambda_s \int_l^\infty y P(y,t) dy}_{\text{splitting}} - \underbrace{\mu_a l P(l,t)}_{\text{attachment}}$$

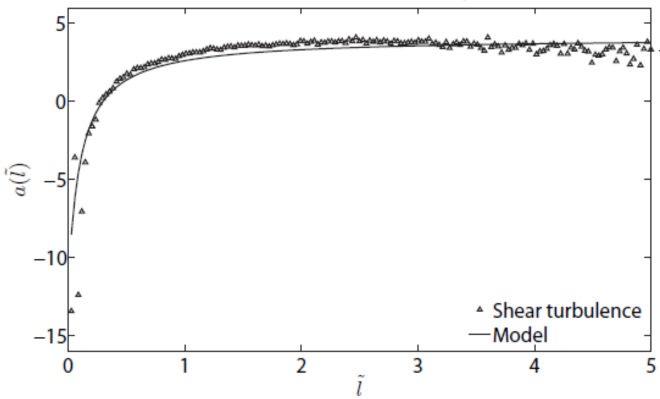
Parameters $D = \nu$ and strain $a(l) \sim 1/\tau$ leads to the **Taylor scale**.

Conditional mean strain rate of dissipation elements

$$\Delta u = a(l) \cdot l$$







a_∞

Scaling with a_∞ instead of $1/\tau$ collapses the normalized velocity increments of different flow configurations.

Derivation of the e - equation by taking appropriate moments of the evolution equation

For homogeneous shear turbulence

$$\frac{\partial \varepsilon}{\partial t} = c_{\varepsilon 1} \, (\overline{-u'v'}) \, \frac{\varepsilon}{k} \frac{\partial \bar{u}}{\partial y} - c_{\varepsilon 2} \frac{\varepsilon^2}{k}$$

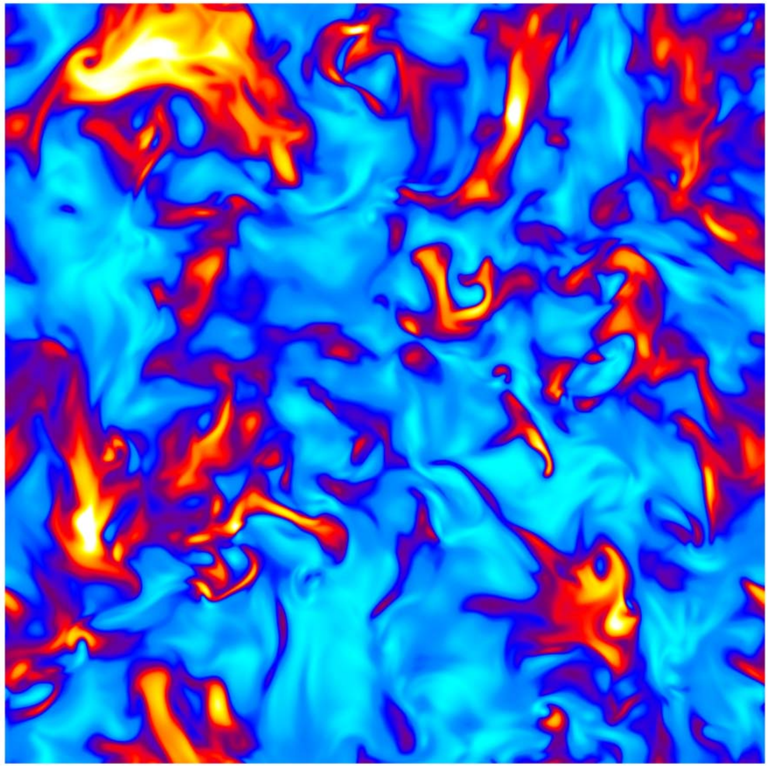
Standard values: $c_{\varepsilon 1} = 1.44$, $c_{\varepsilon 2} = 1.9$

Are the constants (!) $c_{\varepsilon 1}$ and $c_{\varepsilon 2} = 1.9$ Reynolds-number dependent?

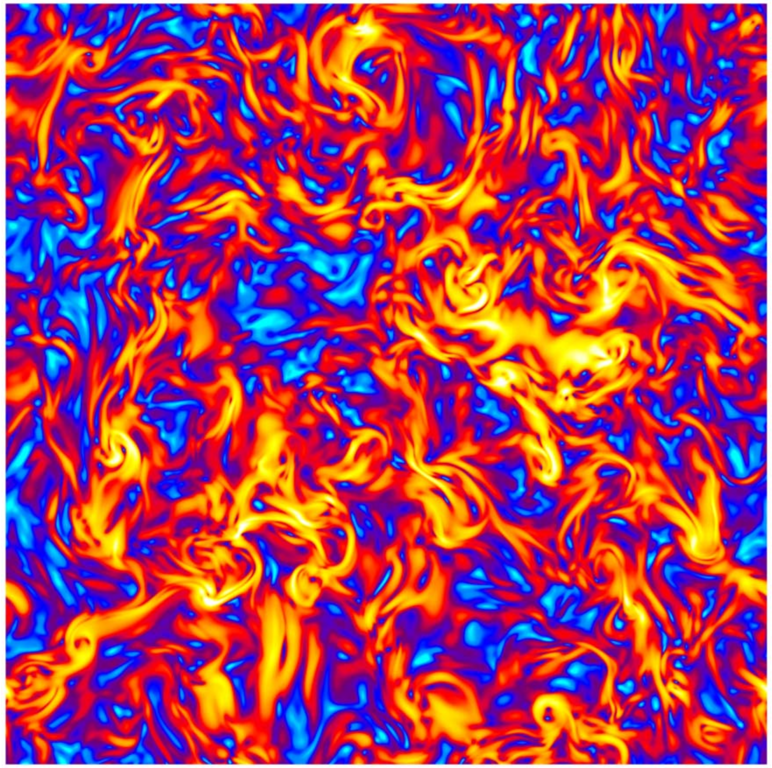
Consider decaying turbulence: $k \sim (t - t_0)^{-m}$, $m = \dfrac{1}{c_{\varepsilon 2 - 1}}$

Experimental data:	$m = 1.25$	$\rightarrow$	$c_{\varepsilon 2} = 1.8$
Final stage, Re $\rightarrow$ 0:	$m = 1.5$ - 2.5	$\rightarrow$	$c_{\varepsilon 2} = 1.66$ - 1.4

Scalar fields of kinetic energy k and dissipation ε



kinetic energy k

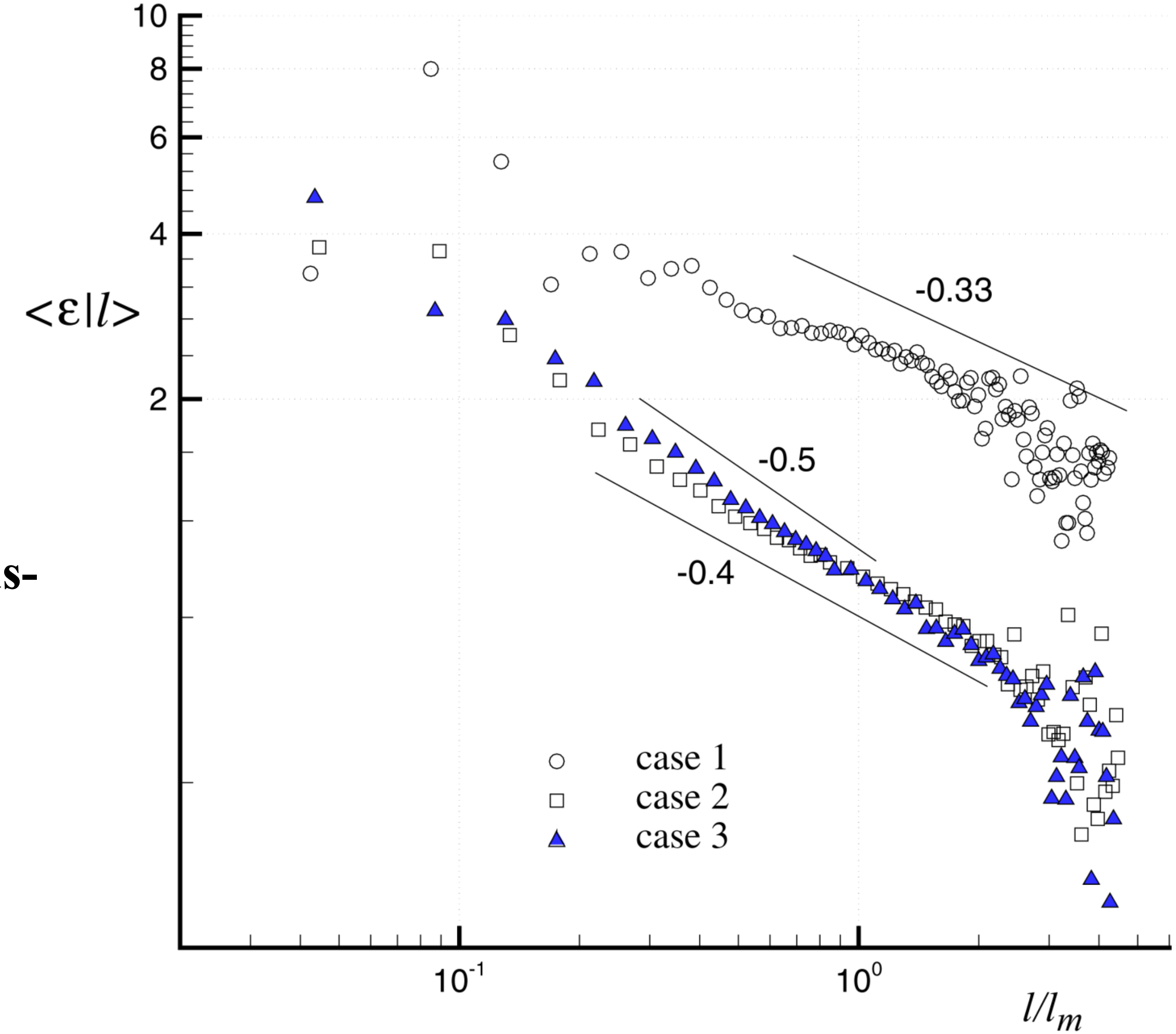


dissipation ε

Starting point:

Conditional dissipation

$$\langle \varepsilon | l \rangle \;=\; \varepsilon^* \, (l/l_m)^n$$



Exponent *n* is Reynolds-number dependent

Relation to ε - equation

$$\frac{\partial \varepsilon}{\partial t} = a_\infty \, \varepsilon^* \big(\underbrace{I_s - I_a - I_{\mathsf{strain}}}_{\text{production}} - \underbrace{I_{\mathsf{drift}}}_{\text{dissipation}} \big)$$

Production term: $$a_\infty \, \varepsilon^* \, I_{\mathsf{prod}} \sim \varepsilon^* \frac{\partial \bar{u}}{\partial y} \sim c_{\varepsilon 1}(\mathsf{Re}) \, (-\overline{u'v'}) \, \frac{\varepsilon}{k} \frac{\partial \bar{u}}{\partial y}$$

If mean length l_m is proportional to the **Taylor scale**: $$l_m^2 \sim \lambda^2 = 10 \, \nu \, \frac{k}{\varepsilon}$$

Dissipation term: $$a_\infty \, \varepsilon^* \, I_{\mathsf{drift}} \sim \varepsilon \, \nu / l_m^2 \sim c_{\varepsilon 2}(\mathsf{Re}) \, \frac{\varepsilon^2}{k}$$

Since n is Reynolds number dependent, so must be $c_{\varepsilon 1}$ and $c_{\varepsilon 2}$.

	case 1	case 2	case 3	Empirical value
Re_λ	98.7	125.0	170.0	–
$c_{\varepsilon 1}$	0.425	0.763	1.20	1.44
$c_{\varepsilon 2}$	0.457	0.923	1.64	1.90

Conclusions

1. **While Kolmogorov's ε – scaling laws tell us how much energy is contained in an element of size *l*, the pdf of linear length provides the additional information on how many elements of size *l* are contained in the flow.**

2. **This pdf equation contains diffusive effects with D=ν and a τ - scaling due to strain and leads to the Taylor scale as mean length scale of dissipation elements.**

3. **Using dissipation elements to reconstruct the ε - equation reproduces the form of the equation but shows a Reynolds number dependence of the empirical modeling constants.**

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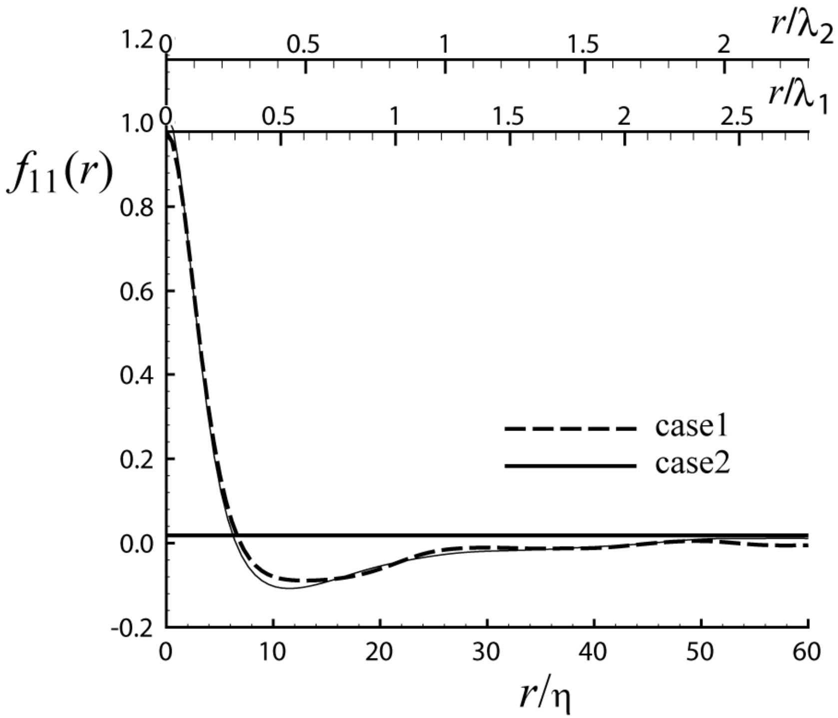
who contributed largely to the recent results.

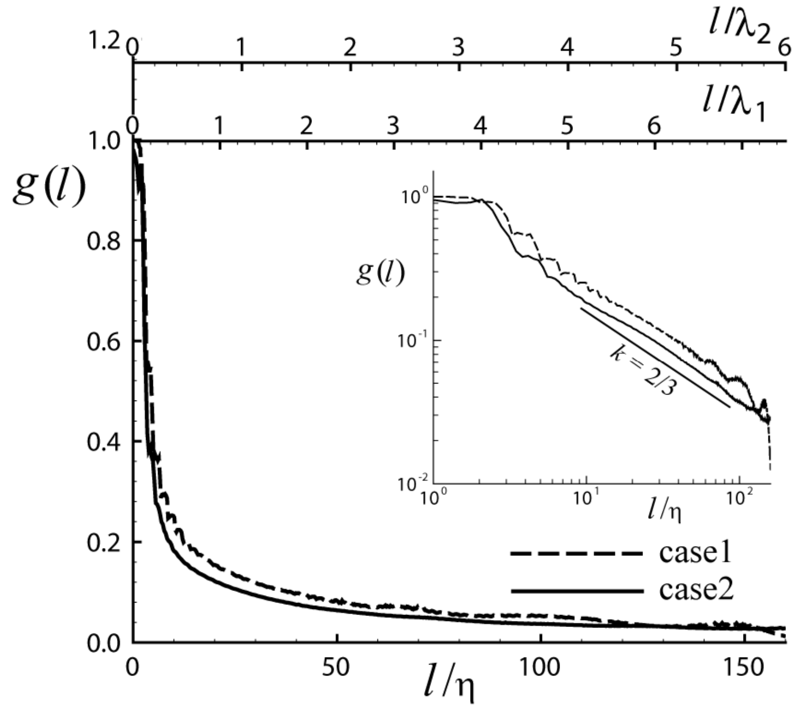
Thank you for your attention

Two point correlation of the scalar gradient

along a straight line

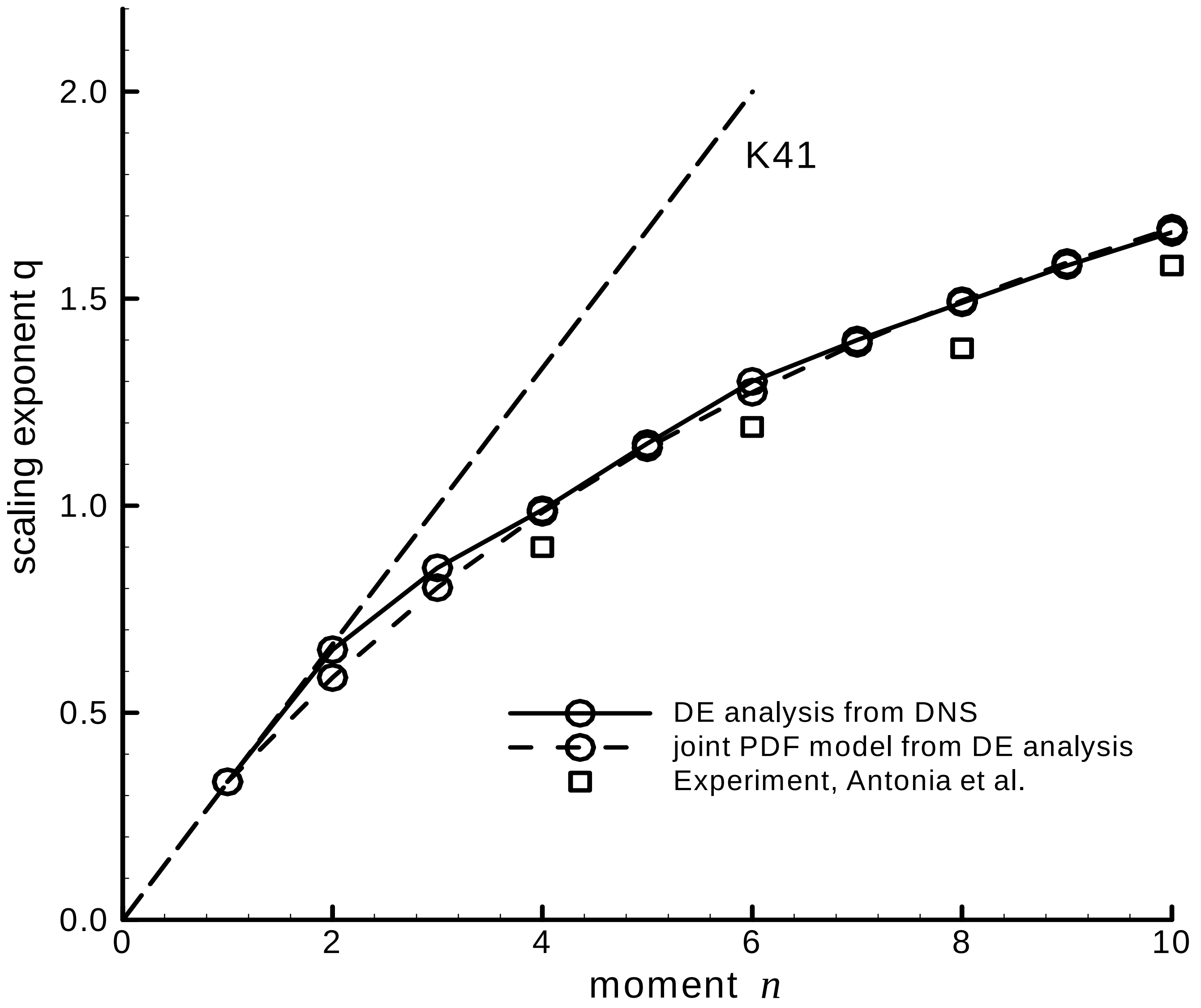


along a gradient trajectory

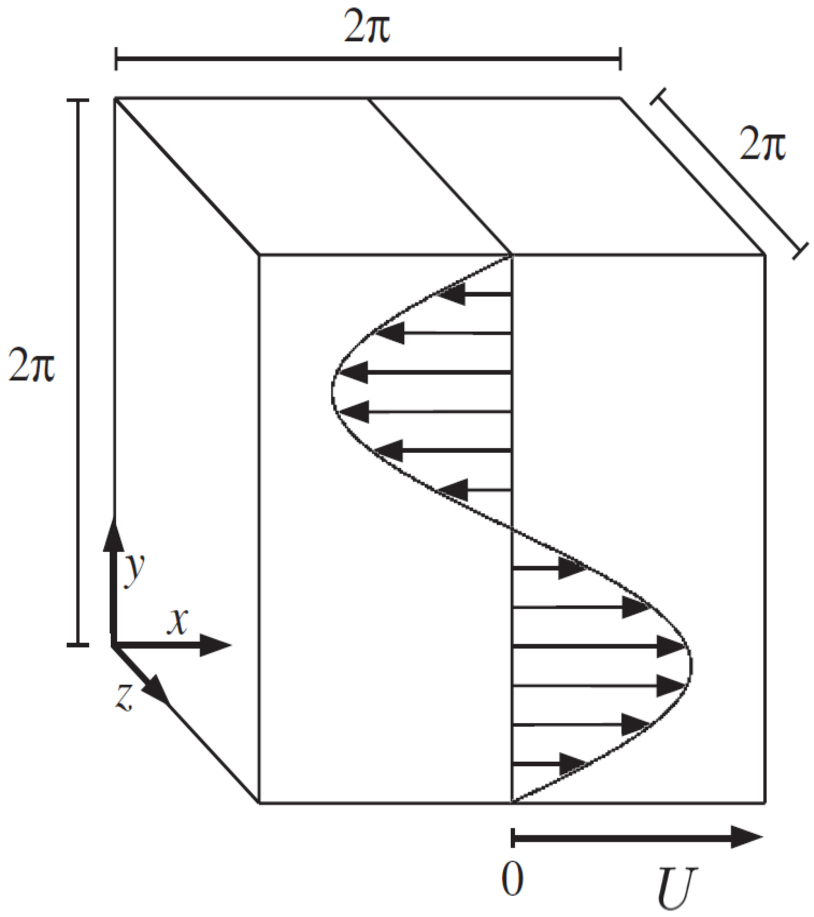


- correlation becomes small for large *l*
- scalar gradient decorrelates from velocity difference

Results: anomalous scaling exponents



Kolmogorov flow



Kolmogorov flow	1024^3	188