Cloud particles in turbulence

R. A. Shaw

Presenting results obtained in collaboration with

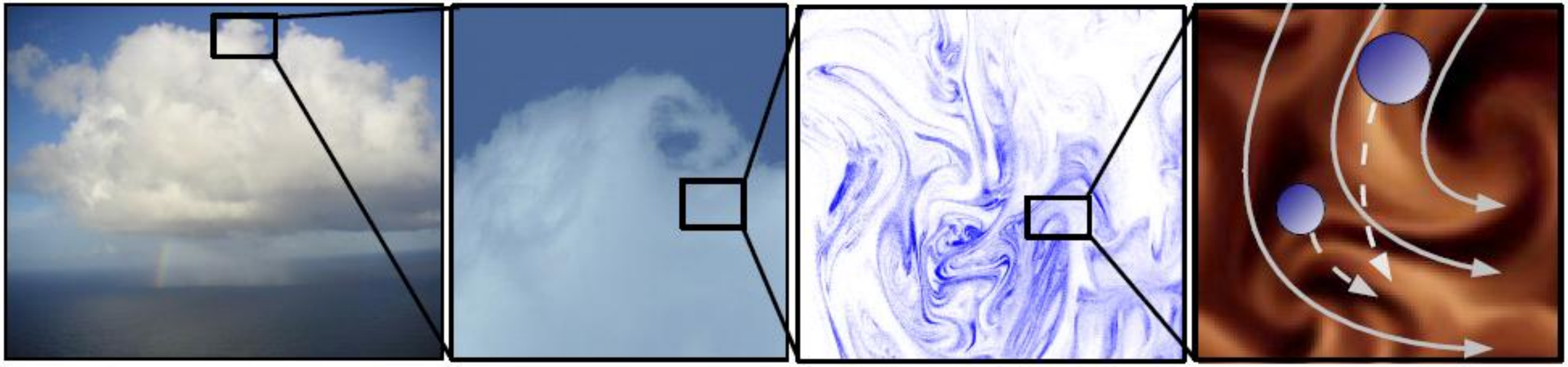
J. Lu – Michigan Technological University

E. W. Saw – Max Planck Institute for Dynamics and Self Organization

J. Schumacher – Technical University Ilmenau

H. Siebert – Leibniz Institute for Tropospheric Research

Also acknowledging colleagues at Cornell University:
L. Collins, J. Salazar, Z. Warhaft



Adapted from Bodenschatz et al. 2010

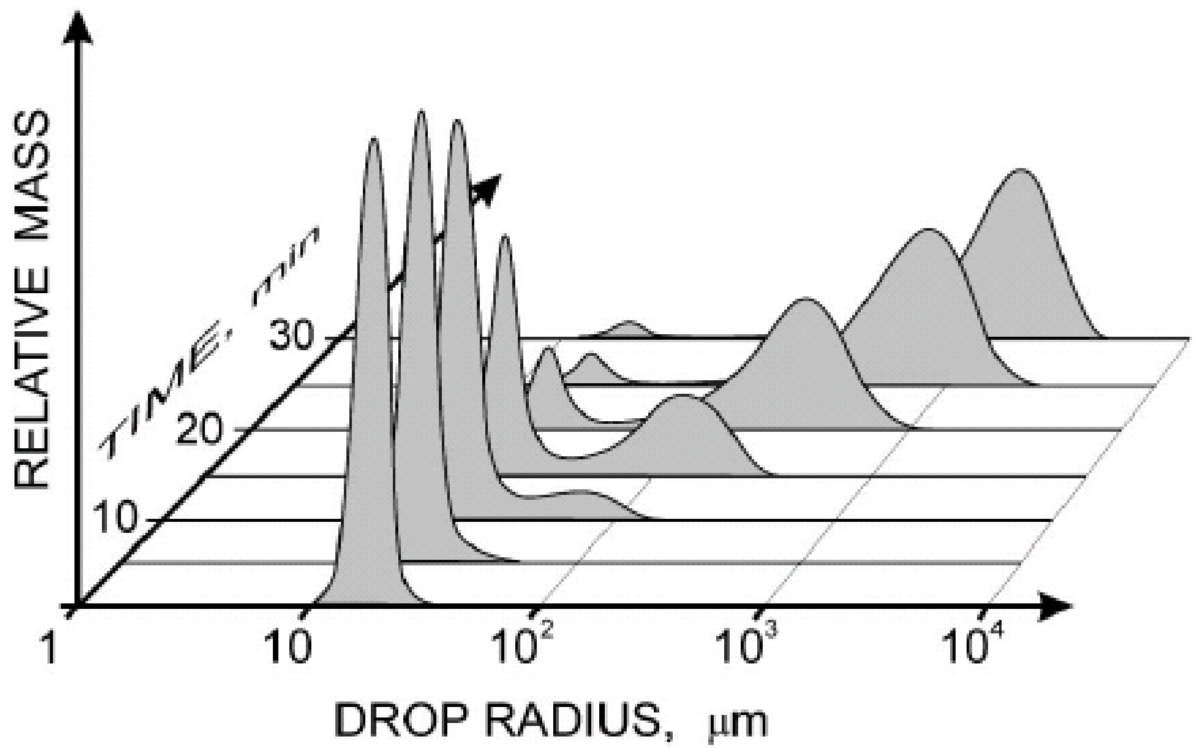
The cloud problem: spatial and temporal evolution of the particle size distribution

The cloud droplet size distribution is defined as $f(r) = np(r)$, where n is the number density of droplets (of all radii) in the volume.

$$
\begin{aligned}
\frac{D' f(r)}{D' t} &= \frac{\partial f(r)}{\partial t} + (\mathbf{v} - \mathbf{k} v_t) \cdot \nabla f(r) \\
&= J - \frac{\partial \left[\dot{r} f(r) \right]}{\partial r} \\
&\quad - \int_0^\infty \kappa\left(r, r'\right) f(r)\, f\left(r'\right) dr' \\
&\quad + \frac{1}{2} \int_0^r \left(\frac{r}{r''}\right)^2 \kappa\left(r'', r'\right) f\left(r''\right) f\left(r'\right) dr'
\end{aligned}
$$

The cloud problem: spatial and temporal evolution of the particle size distribution

The cloud droplet size distribution is defined



Gravitational settling vs. droplet inertia

$$\frac{dv_i}{dt} = -\frac{1}{\tau_p}\left(v_i - u_i\right) + g_i \qquad w_i \equiv v_i - u_i$$

$$\frac{dw_i}{dt} = -\frac{w_i}{\tau_p} - \frac{du_i}{dt} + g_i$$

$$\tilde{t} \equiv \frac{t}{\tau_\eta} \qquad \tilde{w} \equiv \frac{w}{u_\eta} \qquad \mathrm{St} \equiv \frac{\tau_p}{\tau_\eta} \qquad \mathrm{Ac} \equiv \frac{a_\eta}{g}$$

$$\frac{d\tilde{w}_i}{d\tilde{t}} = -\frac{\tilde{w}_i}{\mathrm{St}} - \frac{d\tilde{u}_i}{d\tilde{t}} + \frac{\hat{g}}{\mathrm{Ac}}$$

$$\mathrm{Sv} \equiv \frac{v_T}{u_\eta} = \frac{\tau_p g}{u_\eta} = \frac{\mathrm{St}}{\mathrm{Ac}}$$

turbulent velocity and number density n of droplets are governed by

$$\frac{dv_i}{dt} = \frac{1}{\tau_\mathrm{d}}(u_i - v_i) + g_i, \qquad \frac{\partial n}{\partial t} + v_i \frac{\partial n}{\partial x_i} = -n \frac{\partial v_i}{\partial x_i}, \tag{7.49}$$

with τ_d the droplet inertial time scale resulting from Stokes' solution and v_i the droplet velocity. A negative droplet velocity divergence on the rhs of the second of Eq. (7.49) indicates local congregation of droplets.

For small τ_d we can write the solution of the first of (7.49) as

$$v_i = u_i + \tau_\mathrm{d} g_i - \tau_\mathrm{d} \left(\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} \right) + O(\tau_\mathrm{d}^2), \tag{7.50}$$

so to leading order the droplet velocity divergence is

$$\frac{\partial v_i}{\partial x_i} = -\tau_\mathrm{d} \frac{\partial u_i}{\partial x_j} \frac{\partial u_j}{\partial x_i} = -\frac{\tau_\mathrm{d}}{4} \left(s_{ij} s_{ij} - r_{ij} r_{ij} \right), \tag{7.51}$$

where we have used $u_{i,j} = s_{ij} + r_{ij}$, the sum of strain- and rotation-rate tensors, Eq. (2.73). Equation (7.51) says that strong local vorticity causes positive cloud droplet divergence; weak local vorticity causes negative divergence, droplet congregation, and therefore higher droplet collision rates. The variance of this droplet

Wyngaard 2010

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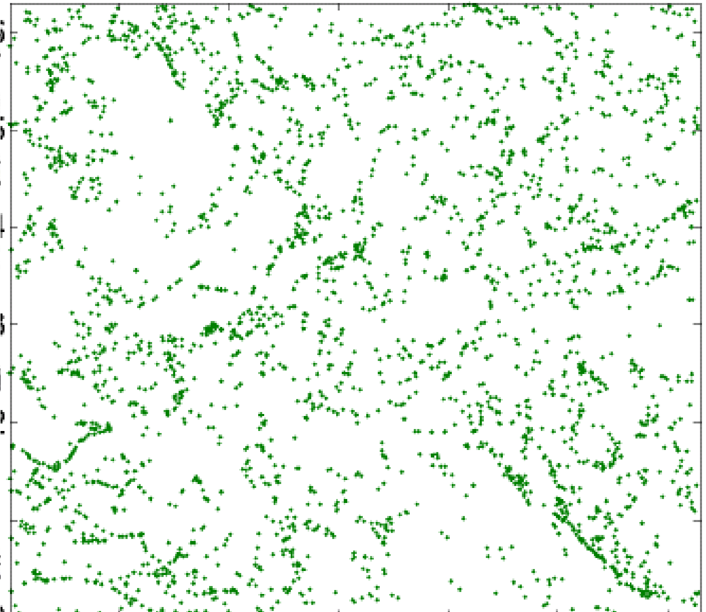
with τ_d the droplet ine … kes' solution and v_i the droplet velocity. A neg … the rhs of the second of Eq. (7.49) indicates loc …



For small τ_d we can … 49) as

$$v_i = \ldots O(\tau_d^2), \tag{7.50}$$

so to leading order the …

$$\frac{\partial v_i}{\partial x_i} = -\tau_d \frac{\partial u_i}{\partial x_j}\frac{\partial u_j}{\partial x_i} = -\frac{\tau_d}{4}\left(s_{ij}s_{ij} - r_{ij}r_{ij}\right), \tag{7.51}$$

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Wyngaard 2010

ACTOS: Airborne Cloud Turbulence Observation System

Siebert, H., H. Franke, K. Lehmann, R. Maser, E. W. Saw, D. Schell, R. A. Shaw, and M. Wendisch: *Bull. Amer. Meteor. Soc.,* 87, 1727-1738 (2006).

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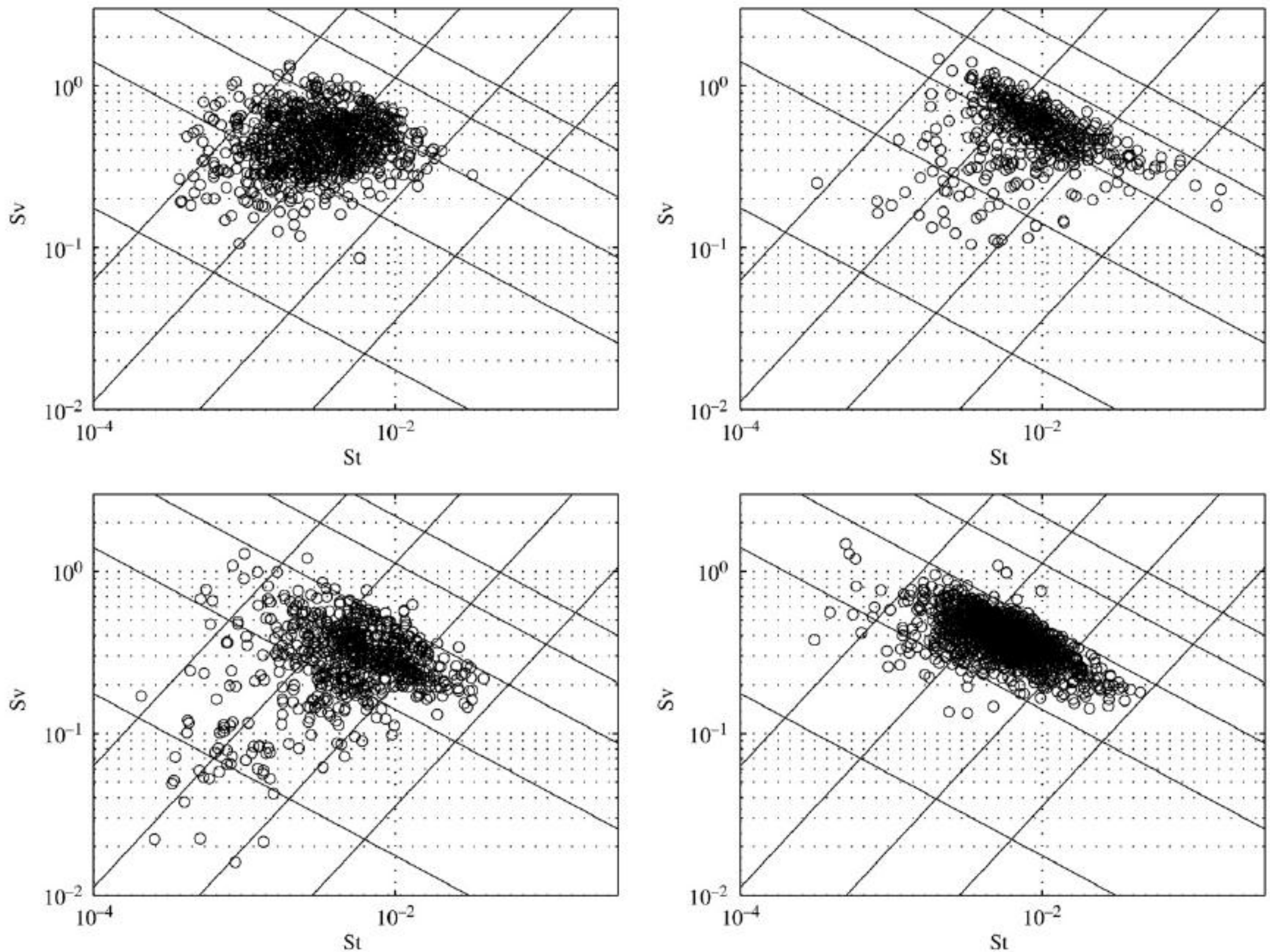
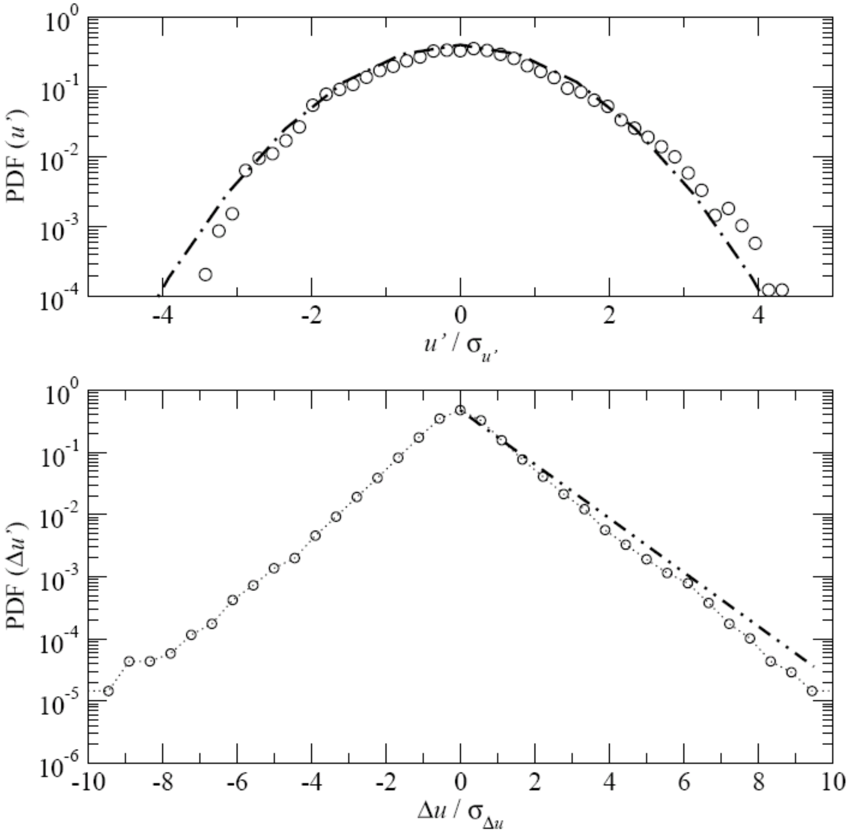
Gravitational settling vs. droplet inertia

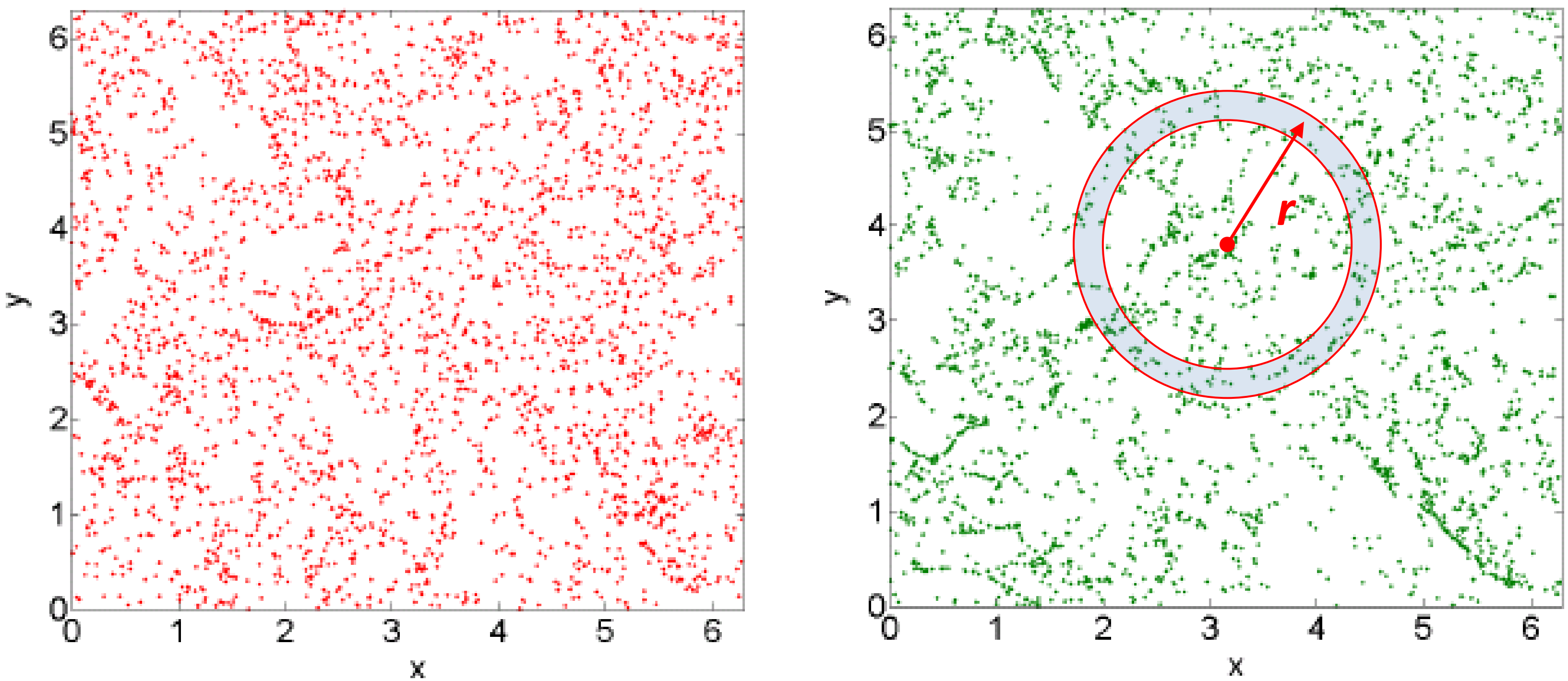


PDFs of velocity and increments in stratocumulus cloud



Siebert, H., R. A. Shaw, Z. Warhaft, J. Atmos. Sci., 2010

Inertial clustering & radial distribution function



$$g(r) = \frac{\psi(r)/N}{(N-1)\,\delta V_r/V} \qquad g_{ij}(r) = \frac{\psi_{ij}(r)/N_i}{N_j\,\delta V_r/V}$$

$$g(r, a \leqslant St \leqslant b) = \int_a^b \int_a^b g_{ij}(r, St_i, St_j)\,\rho(St_i)\,\rho(St_j)\,dSt_i\,dSt_j$$

Drift-diffusion theory for clustering…
(Chun et al. … similar in spirit to Falkovich et al., Zaichik & Alipchenkov)

$$0 = -\langle w \rangle_{\mathrm{p}} g_{12} + \mathcal{D}_{12} \frac{\mathrm{d}g_{12}}{\mathrm{d}r}$$

from Fokker-Planck eqn for particle pair probability *g(r)*
… steady state

$$\langle w \rangle_{\mathrm{p}} = -\frac{St_2}{3\tau_\eta} \left[\langle \mathcal{S}^2 \rangle_{\mathrm{p}} - \langle \mathcal{R}^2 \rangle_{\mathrm{p}} \right] r$$

$$\mathcal{D}_{12} = \left(\frac{B_{\mathrm{nl}}}{\tau_\eta} \right) r^2 + \mathcal{D}_{\|}$$

$$\mathcal{D}_{\|}^{\mathrm{a}} = \left(\tau_{\mathrm{p}}^{[2]} - \tau_{\mathrm{p}}^{[1]} \right)^2 a_0 \, a_\eta^2 \, \tau_{\mathrm{a}}$$

monodisperse:

$$g(r) = c_0 \left(\frac{\eta}{r} \right)^{c_1} \qquad c_1 = \frac{\langle w \rangle_p}{v_{\mathrm{diff}}}$$

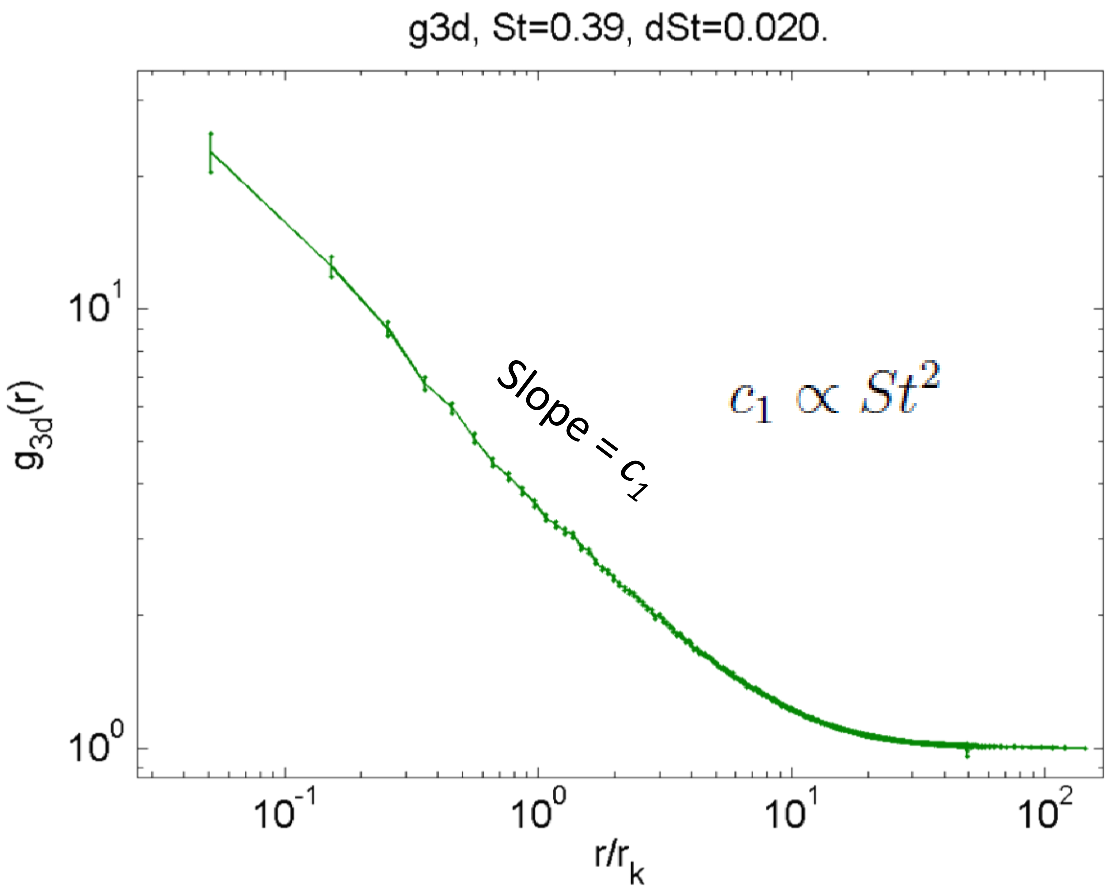
bidisperse:

$$g_{12}(r) = c_0 \left(\frac{\eta^2 + r_{\mathrm{c}}^2}{r^2 + r_{\mathrm{c}}^2} \right)^{c_1/2}$$

$$\left(\frac{r_{\mathrm{c}}}{\eta} \right)^2 = \frac{1}{B_{\mathrm{nl}}} \left(\frac{\tau_{\mathrm{a}}}{\tau_\eta} \right) \left[a_0 \left(St_2 - St_1 \right)^2 \right.$$

Theoretical prediction for RDF: Power law

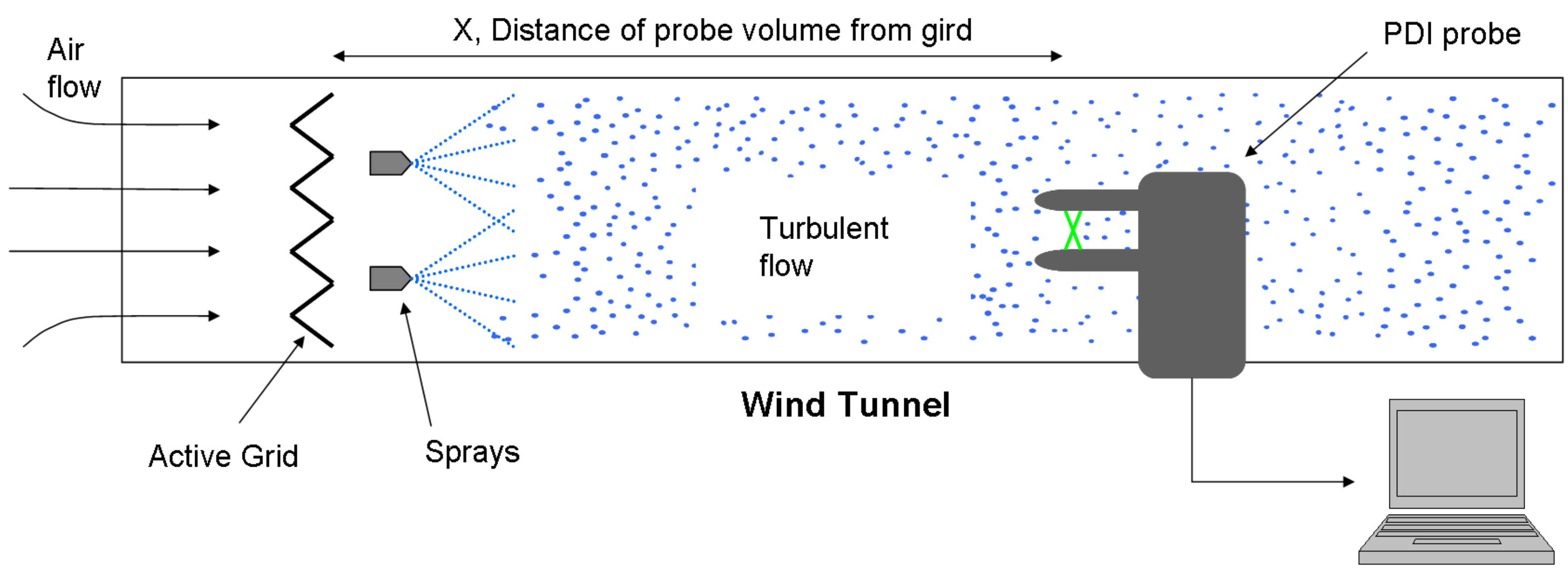
$$g(r) = c_0 \left(\frac{r_k}{r} \right)^{c_1} \qquad for \quad r < r_k \,, \quad St << 1$$



r_k = Kolmogorov length

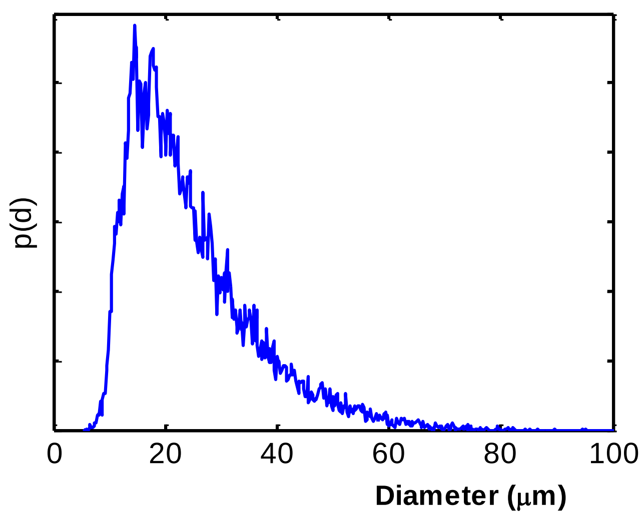
Saw et al. 2011

Active-grid wind tunnel with spray



Cornell active-grid wind tunnel: Variable flow speed U and position of probe X.

- Length 15 m, Cross section 1 m^2
- Longitudinal mean speed 1-10 m/s
- rms speed ~ 10% mean
- R_λ = 300-900
- Mean diameter ~ 20 μm



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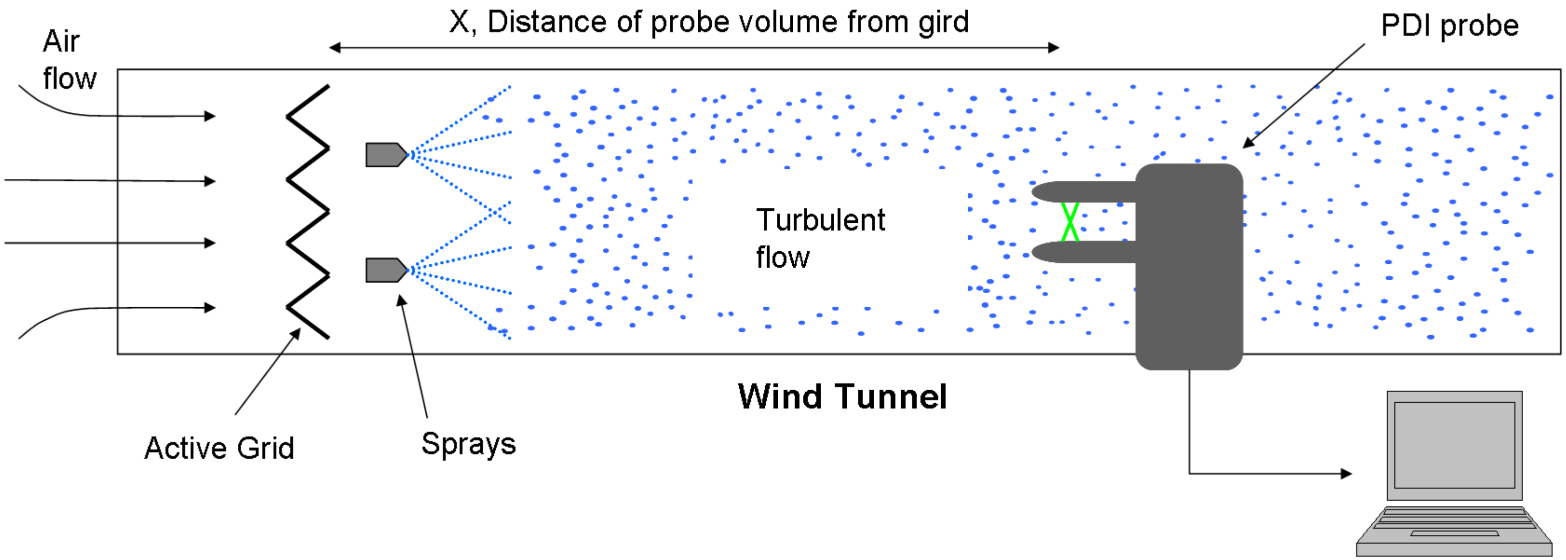

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- Length 15 m, Cross section 1 m^2
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- R_λ = 300-900
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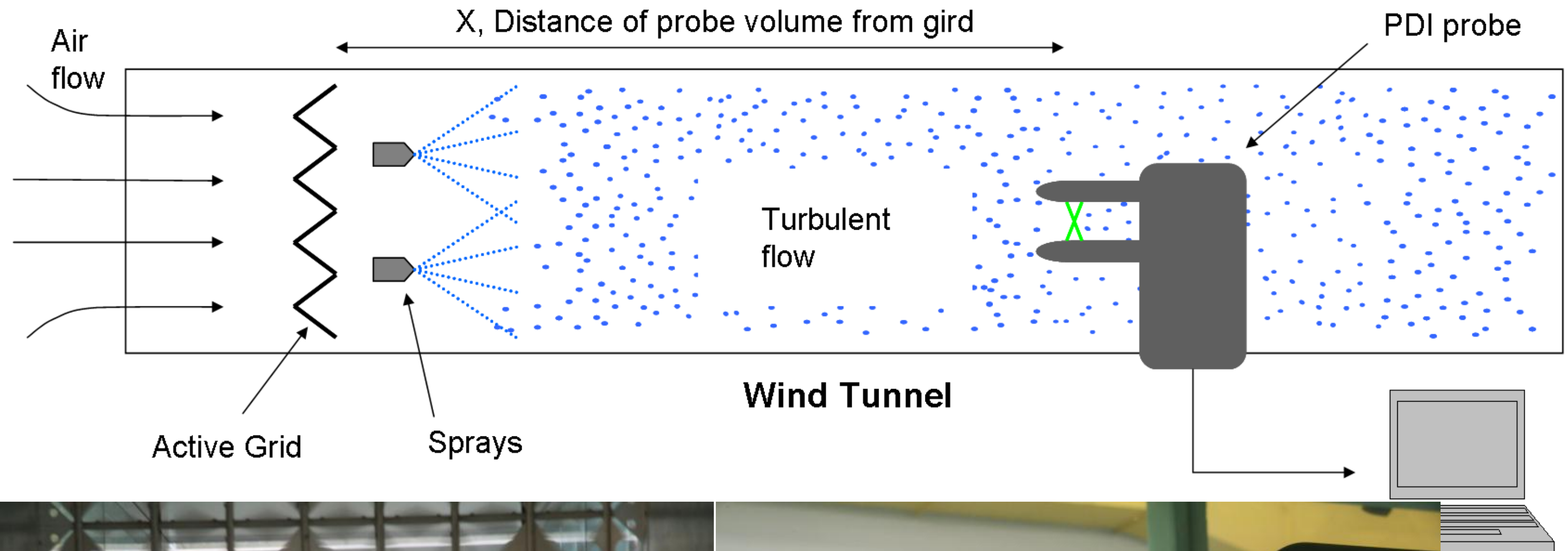
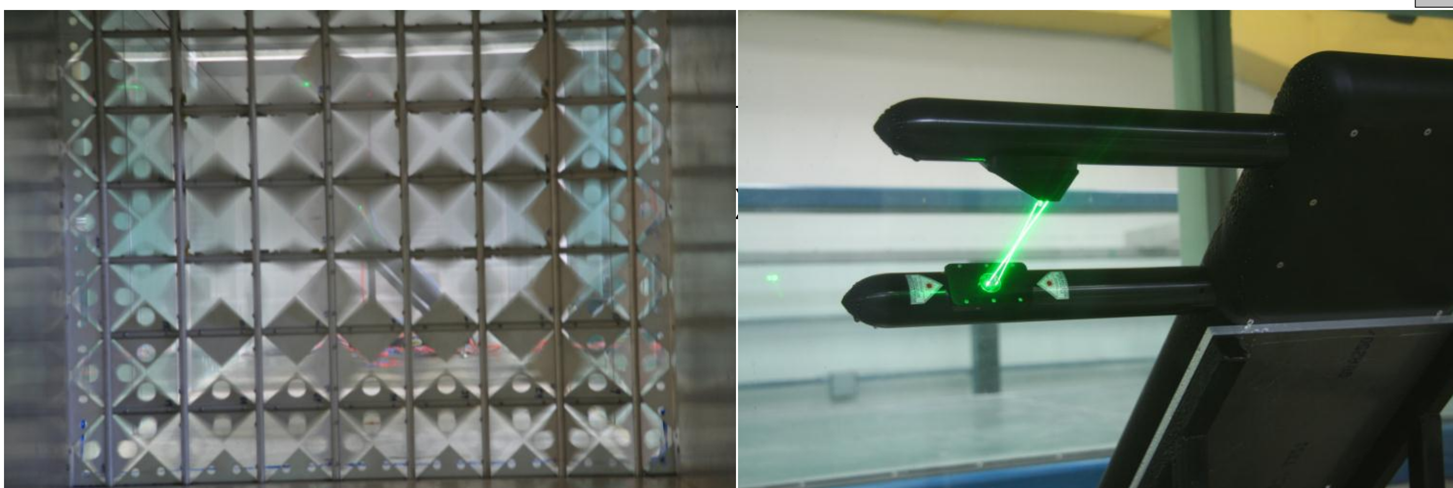
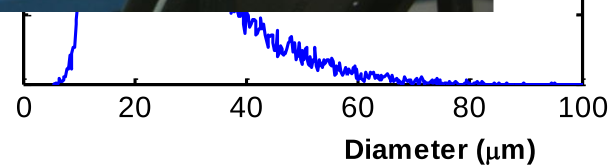
$$St = \frac{\tau_d}{\tau_k} = \frac{1}{18}\left(\frac{\rho_d}{\rho}\right)\left(\frac{d}{r_k}\right)^2$$

Active-grid wind tunnel with spray

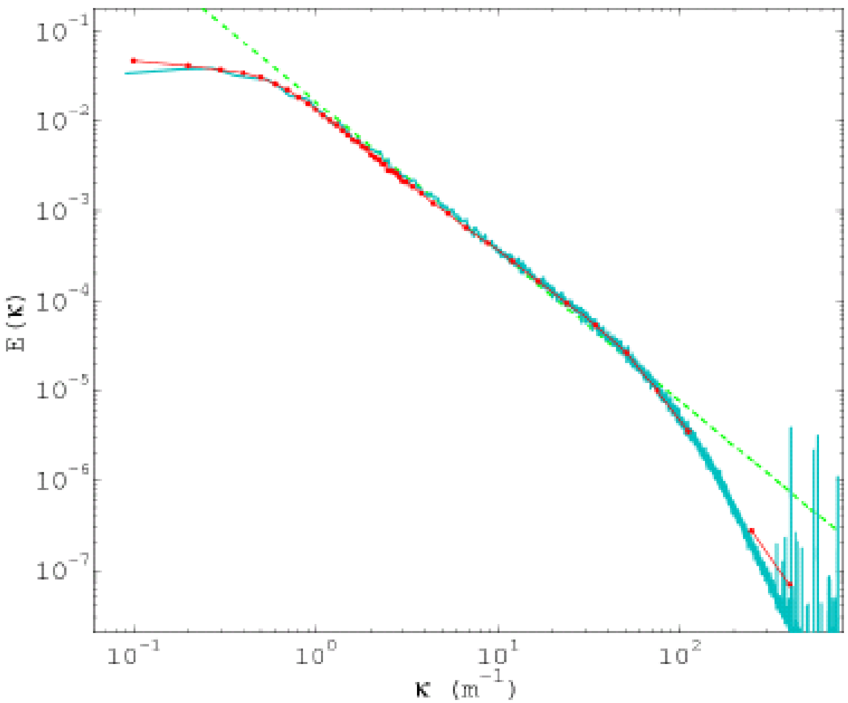


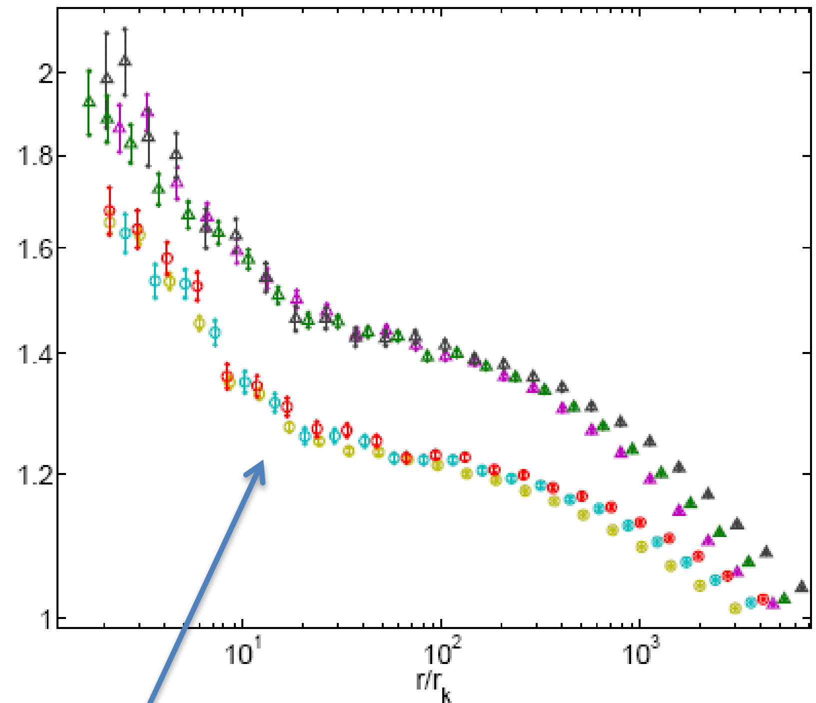




- R_λ = 300-900
- Mean diameter ~ 20 μm

Turbulence energy spectrum and RDF…

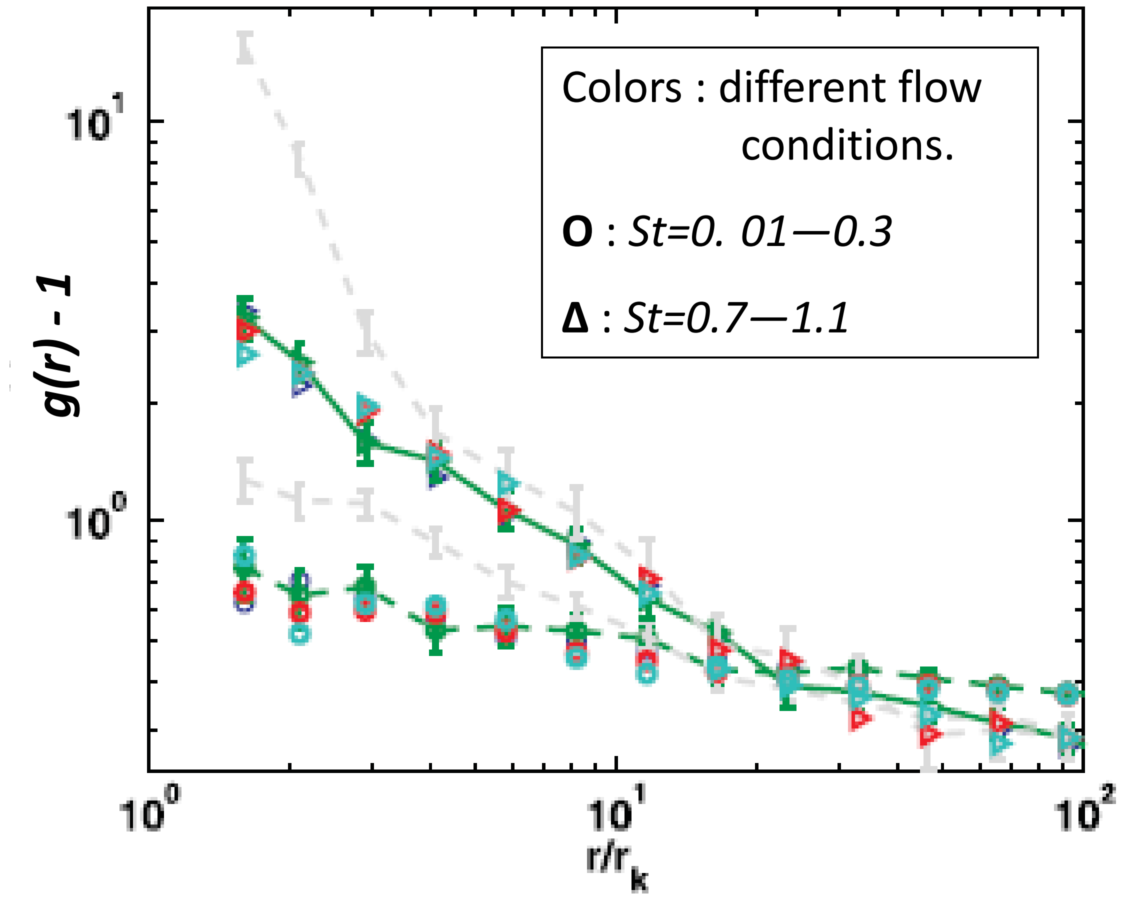




Inertial clustering scale break

Saw et al. 2011

Stokes number scaling…

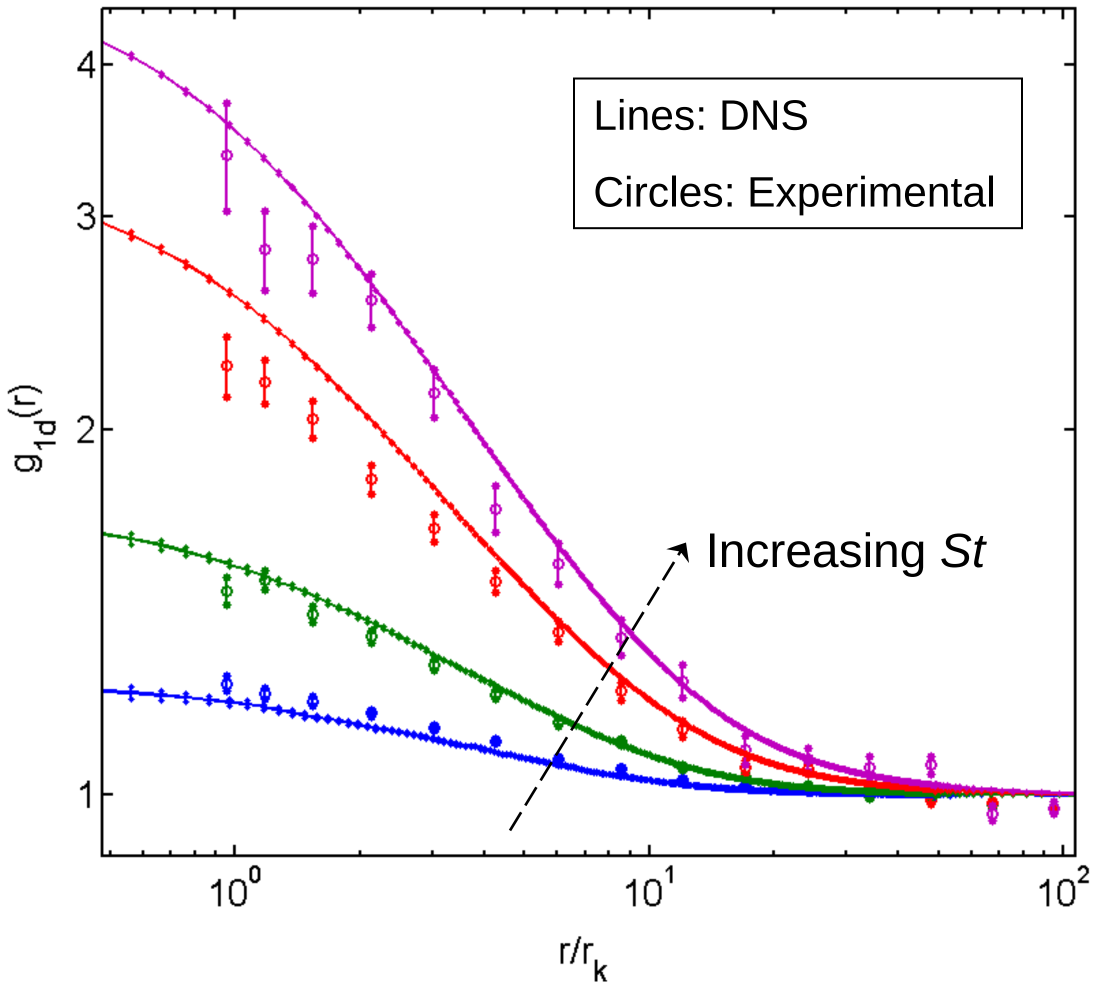


- Results from different flow conditions coincide when *St* are matched.
- *St* effects dominate over gravity, *Re* effects.
- For triangles, *Sv* goes from 0.2 – 1.0 .
- R_λ= 440—800 .

$$St = \frac{\tau_d}{\tau_k} = \frac{1}{18}\left(\frac{\rho_d}{\rho}\right)\left(\frac{d}{r_k}\right)^2$$

Saw et al. (2008) PRL

Comparison: Experiments & DNS



- **Polydisperse**
- **Reasonable agreement, but small deviation at large St.**

$$g(r, a \le St \le b) = \iint_{a \le St \le b} g_{12}(r, St_1, St_2)\rho(St_1)\rho(St_2)\, dSt_1 dSt_2$$

Saw et al. 2011

$$St = \frac{\tau_p}{\tau_k} = \frac{1}{18}\left(\frac{\rho_p}{\rho}\right)\left(\frac{d}{r_k}\right)^2$$

Drift-diffusion theory for clustering… with gravity

$$0 = -\langle w \rangle_{\mathrm{p}} g_{12} + \mathcal{D}_{12} \frac{\mathrm{d}g_{12}}{\mathrm{d}r}$$

$$\langle w \rangle_{\mathrm{p}} = -\frac{St_2}{3\tau_\eta}\left[\langle \mathcal{S}^2 \rangle_{\mathrm{p}} - \langle \mathcal{R}^2 \rangle_{\mathrm{p}}\right] r$$

$$\mathcal{D}_{12} = \left(\frac{B_{\mathrm{nl}}}{\tau_\eta}\right) r^2 + \mathcal{D}_{\|}$$

$$\mathcal{D}_{\|} = (St_2 - St_1)^2 \left[\left(\frac{a_0\,\eta^2}{\tau_\eta}\right)\left(\frac{\tau_{\mathrm{a}}}{\tau_\eta}\right) + \left(\frac{g^2 \tau_\eta^3}{3}\right)\left(\frac{\tau_{\mathrm{g}}}{\tau_\eta}\right)\right]$$

$$g(r) = c_0 \left(\frac{\eta}{r}\right)^{c_1}$$

$$g_{12}(r) = c_0 \left(\frac{\eta^2 + r_{\mathrm{c}}^2}{r^2 + r_{\mathrm{c}}^2}\right)^{c_1/2}$$

$$\left(\frac{r_{\mathrm{c}}}{\eta}\right)^2 = \frac{1}{B_{\mathrm{nl}}}\left(\frac{\tau_{\mathrm{a}}}{\tau_\eta}\right)\left[a_0\,(St_2 - St_1)^2 + \frac{1}{3}(\mathrm{Sg}_2 - \mathrm{Sg}_1)^2\right]$$

Lu et al. New J. Phys. 2010

Drift-diffusion… with charge
(so we can change "effective" gravity)

$$\langle w \rangle_{\mathrm{p}} = -\frac{St_2}{3\tau_\eta}[\langle \mathcal{S}^2 \rangle_{\mathrm{p}} - \langle \mathcal{R}^2 \rangle_{\mathrm{p}}]r + Ct_{12}\, u_\eta \left(\frac{\eta}{r}\right)^2$$

$$Ct_{12} \equiv \frac{u_{\mathrm{q}}}{u_\eta}$$

$$g(r) = c_0 \left(\frac{\eta}{r}\right)^{c_1}$$

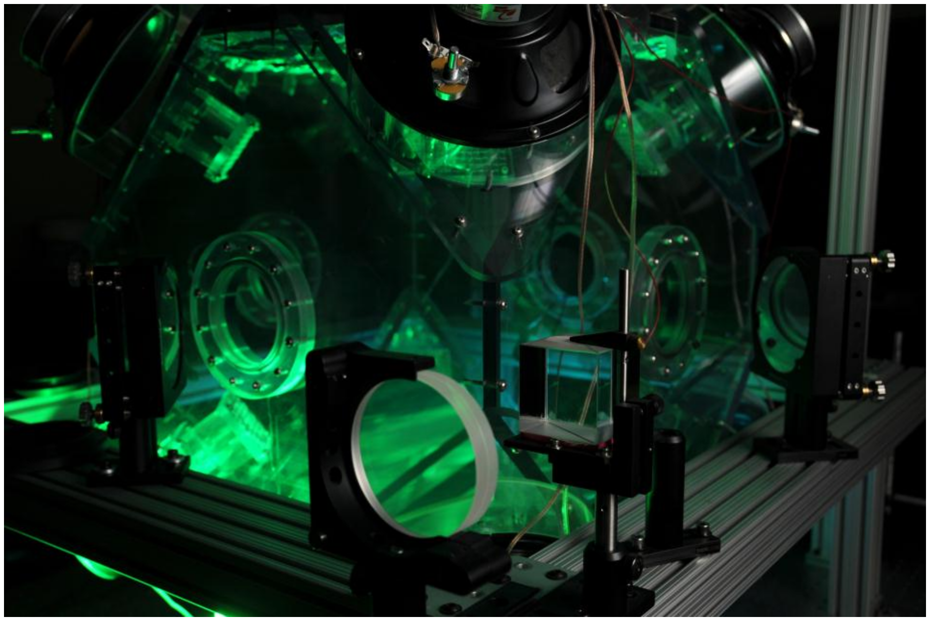
$$g(r) = c_0 \left(\frac{\eta}{r}\right)^{c_1} \exp\left[-c_{2,\mathrm{mono}}\, Ct \left(\frac{\eta}{r}\right)^3\right]$$

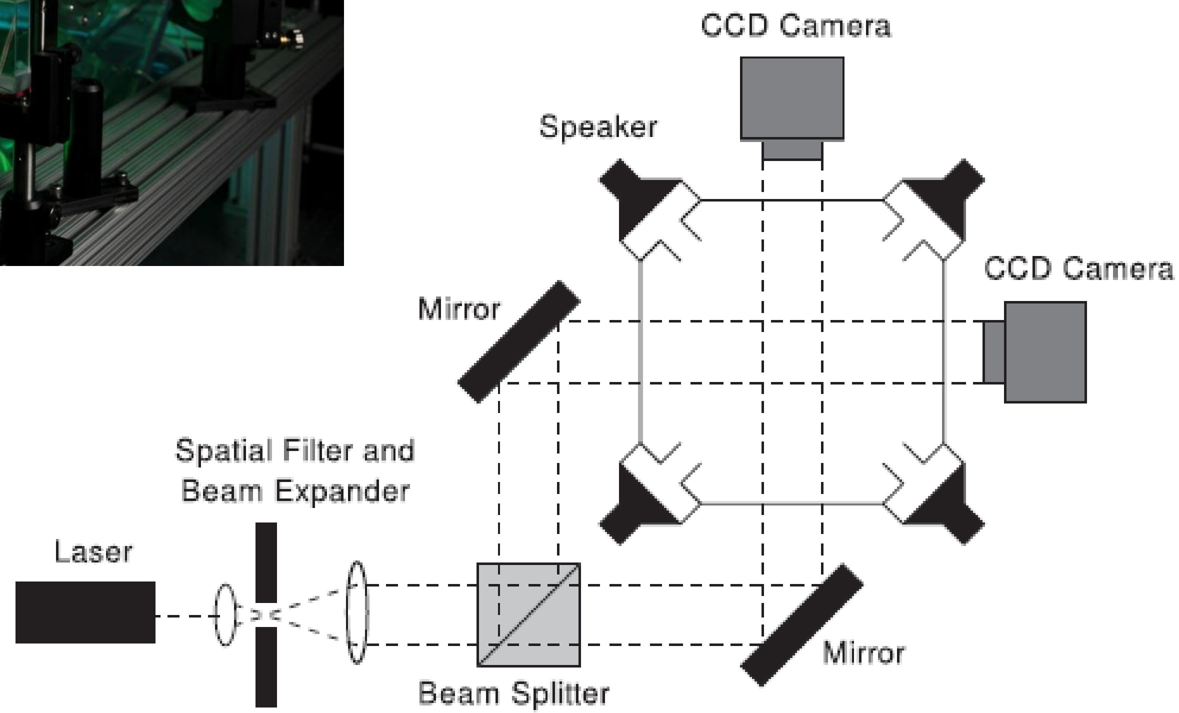
$$Ct \equiv \left(2\,\frac{kq^2}{\eta^2}\right)\left(\frac{u_\eta}{\beta}\right)^{-1}$$

$$\exp(-2v_{\mathrm{charge}}/3v_{\mathrm{turb}})$$

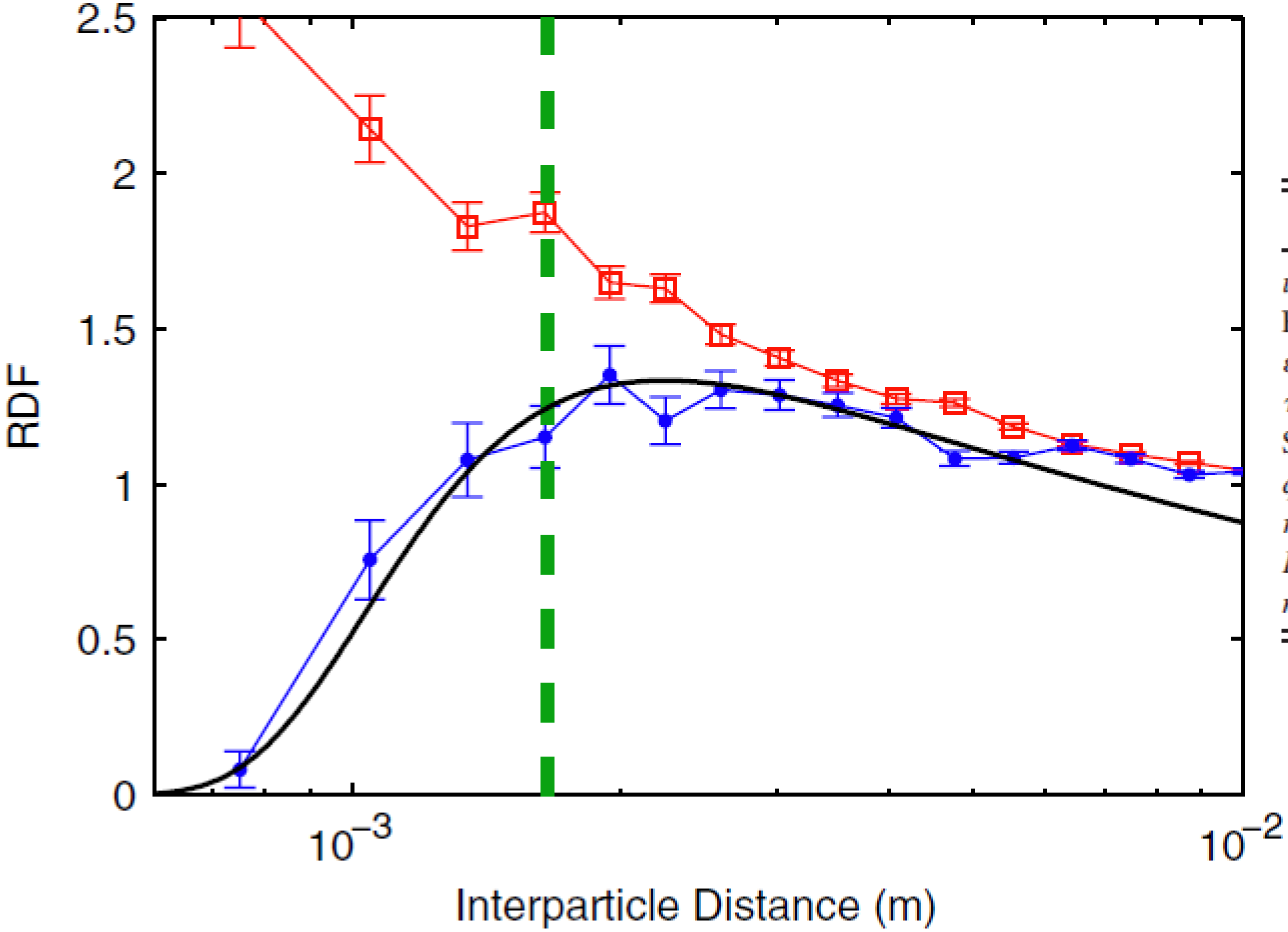
Lu et al. New J. Phys. 2010

Experiment: 3D particle positions in homogeneous, isotropic turbulence…





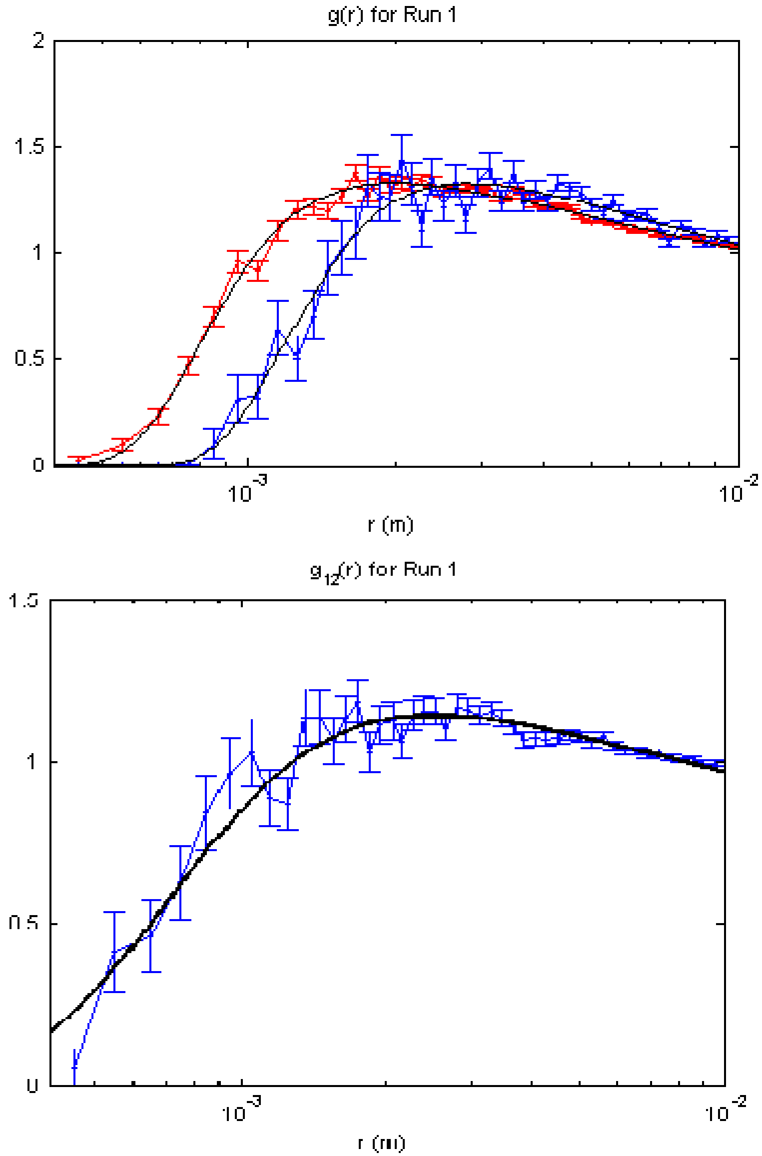
Charged inertial particles in turbulence…



	1c	1n
u_{rms} (cm s^{-1})	15	14
Re_λ	84	82
ε (W kg^{-1})	0.06	0.06
η (μm)	510	510
St	0.3	0.3
q (e)	380 000	~80 000
r_* (mm)	1.7	~0.6
E_{charge}/E_{turb}	2.0	~0.08
$n^{-1/3}$ (mm)	10	7

Lu et al. New J. Phys. 2010

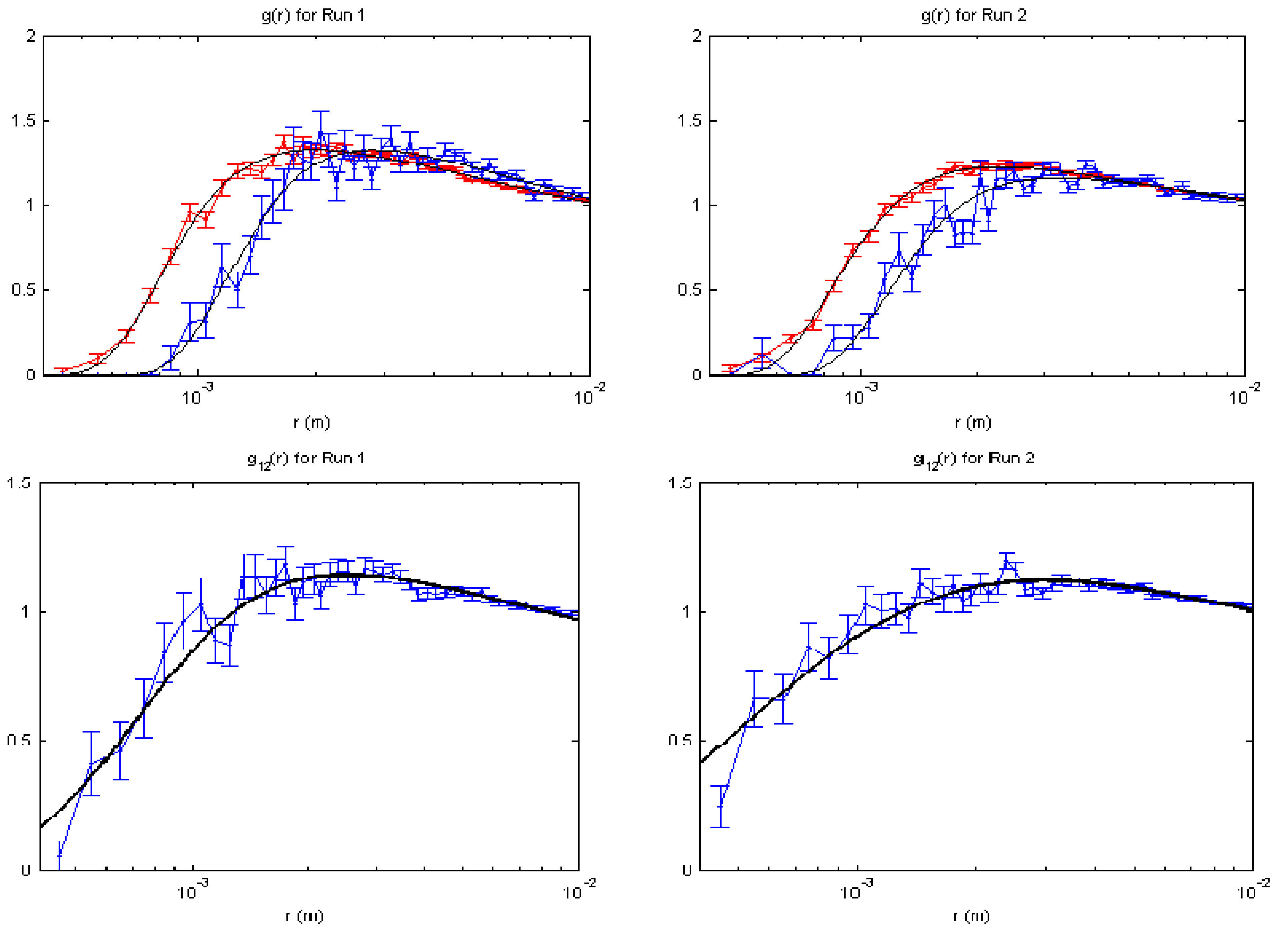
Effect of gravitational settling…



	Run 1	Run 2
g_e (G)	1	1.9
$n^{-1/3}$ (mm)	9.5	7.8
d_1 (μm)	35	35
d_2 (μm)	44	44
q_1 (e)	200 000	200 000
q_2 (e)	390 000	410 000
St_1	0.14	0.14
St_2	0.22	0.22
Sg_1	1.6	3.0
Sg_2	2.5	4.7
Ct_1	0.31	0.34
Ct_2	0.99	1.07
Ct_{12}	0.56	0.60

Lu et al. New J. Phys. 2010

Effect of gravitational settling…



Lu et al. New J. Phys. 2010

The cloud problem, part 2: coupling of turbulence and droplets via thermodynamics

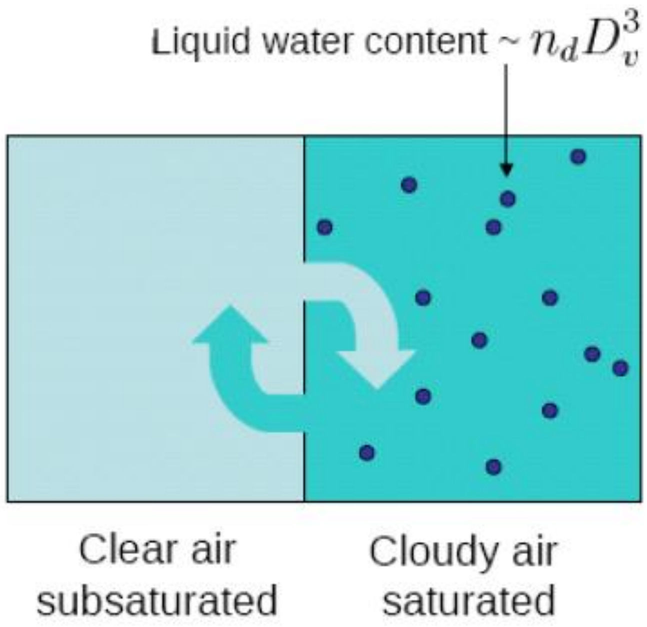
An approximate (Boussinesq) set of equations for cloud velocity $\mathbf{v}$, temperature T, and water vapor mixing ratio q_v:

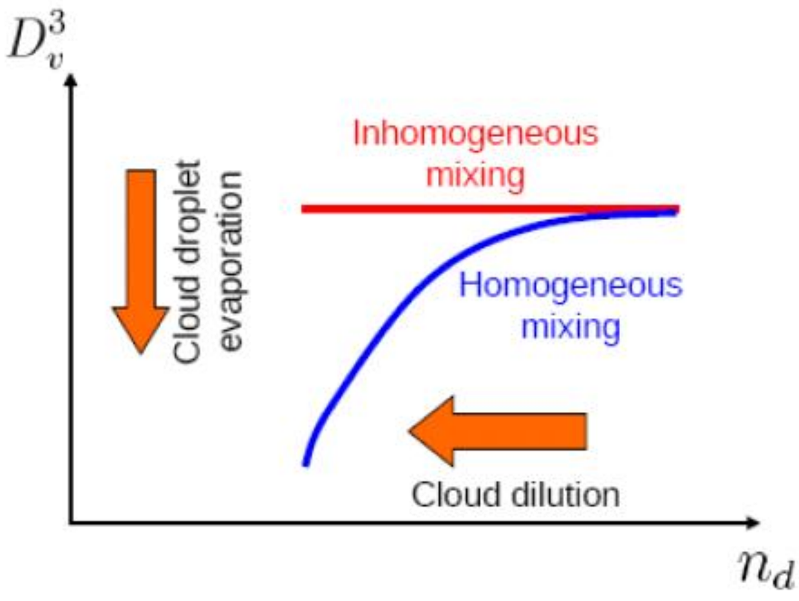
$$\begin{aligned}
\nabla \cdot \mathbf{v} &= 0 \\
\frac{D\mathbf{v}}{Dt} &= -\nabla \frac{p'}{\rho_0} + \mathbf{k}\beta + \nu \nabla^2 \mathbf{v} \\
\frac{DT}{Dt} &= \frac{L}{c_p}\gamma_d + \alpha_T \nabla^2 T \\
\frac{Dq_v}{Dt} &= -\gamma_d + \alpha_v \nabla^2 q_v
\end{aligned}$$

$$\gamma_d = \int_0^\infty \dot{m} f \, dr$$

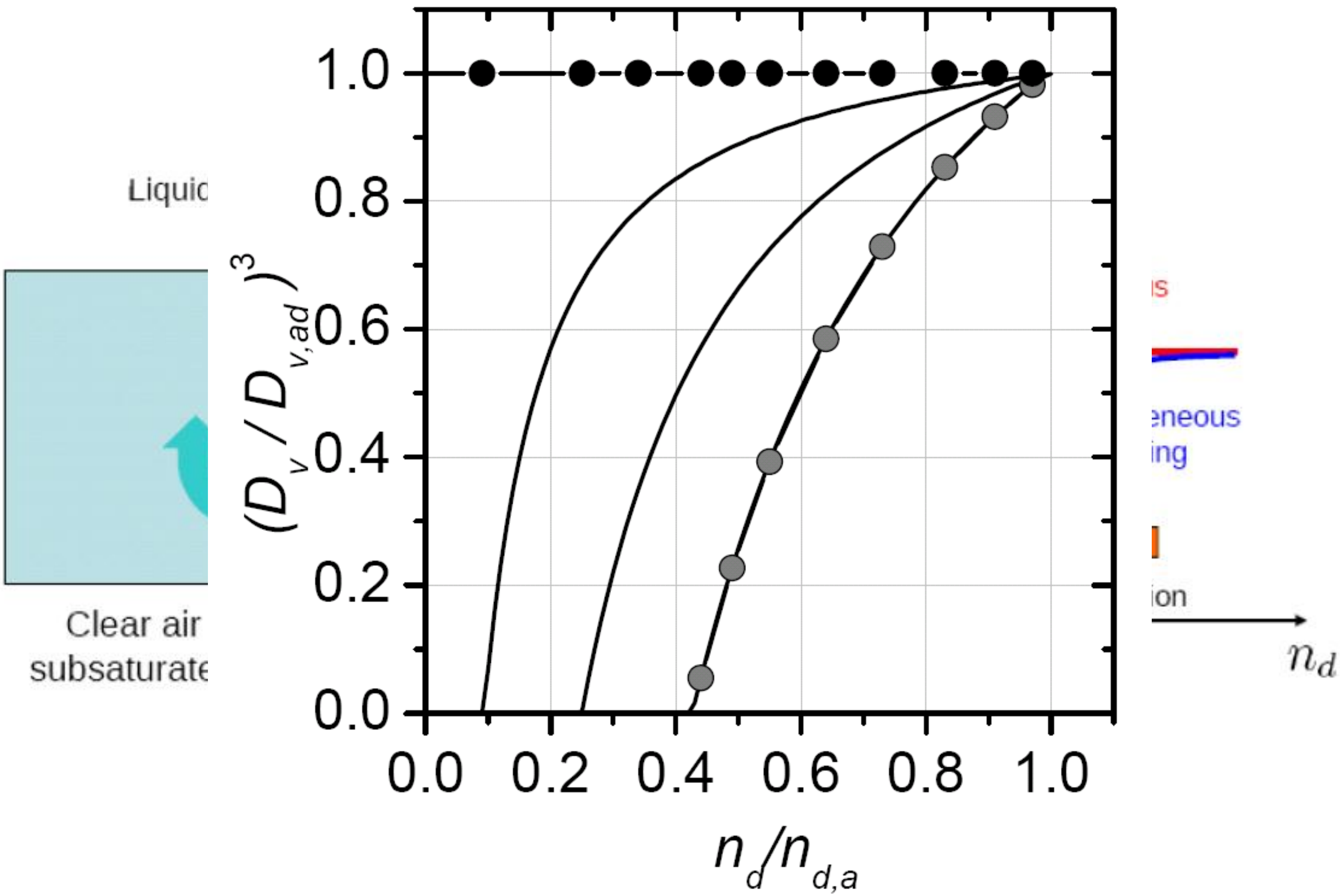
$$\beta \equiv g \left(\frac{T - T_0}{T_0} + \epsilon (q_v - q_{v0}) - \rho_0^{-1} \int_0^\infty m f \, dr \right)$$

How do clouds mix and entrain?





How do clouds mix and entrain?



Turbulent mixing and microphysical time scales

- Mixing characterized by relative values of mixing and evaporation time scales (Damköhler number):

$$Da = \frac{\tau_{mix}}{\tau_{react}}$$

- *Da* << 1: Mixing faster than reaction → homogeneous
- *Da* >> 1: Mixing slow compared to reaction → inhomogeneous

- Mixing timescale from inertial subrange scaling:

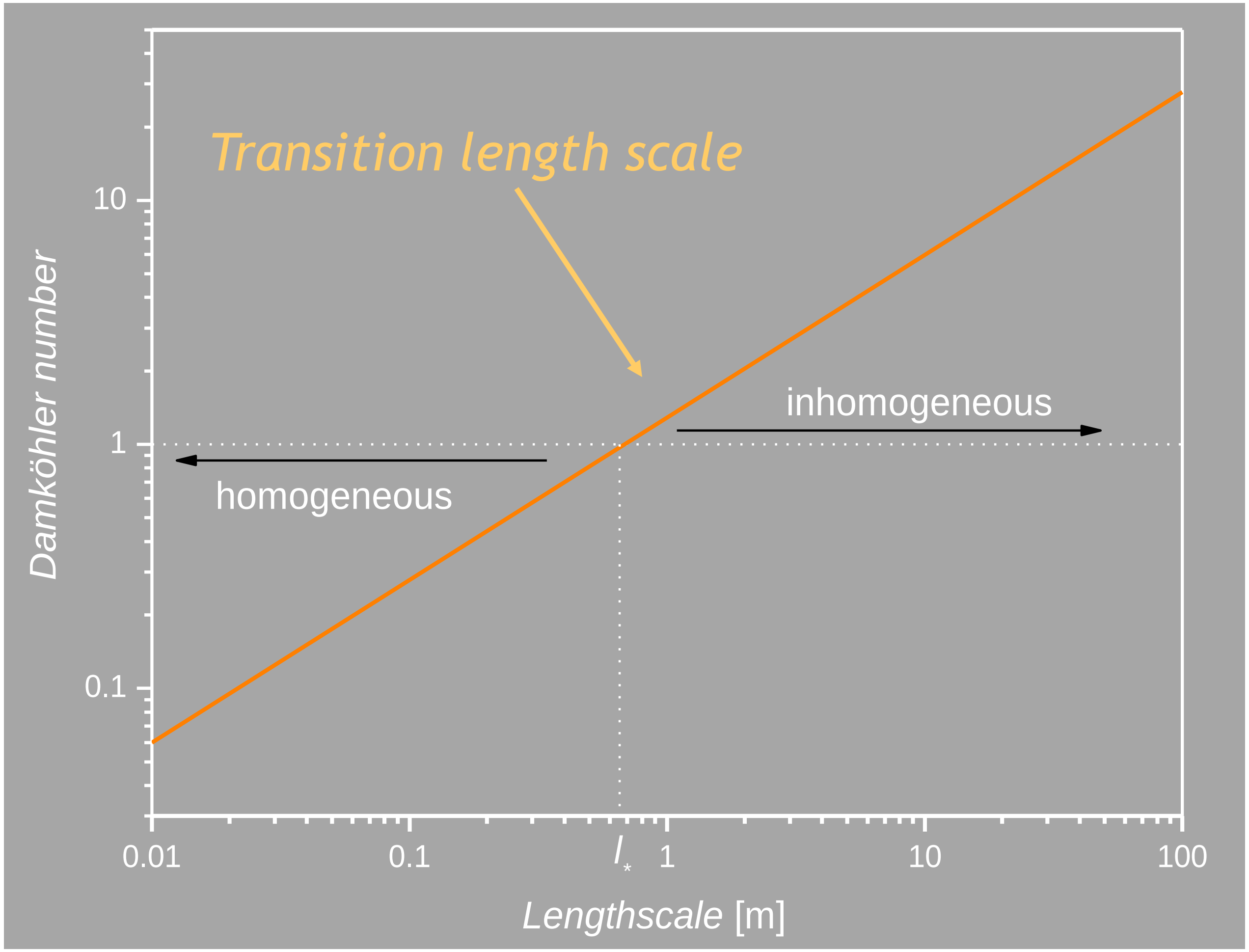
$$\tau_{mix} = \left(\frac{L^2}{\varepsilon} \right)^{1/3}$$

Lengthscale dependence of Damköhler number

$$Da = L^{2/3}\varepsilon^{-1/3}\tau_r^{-1}$$

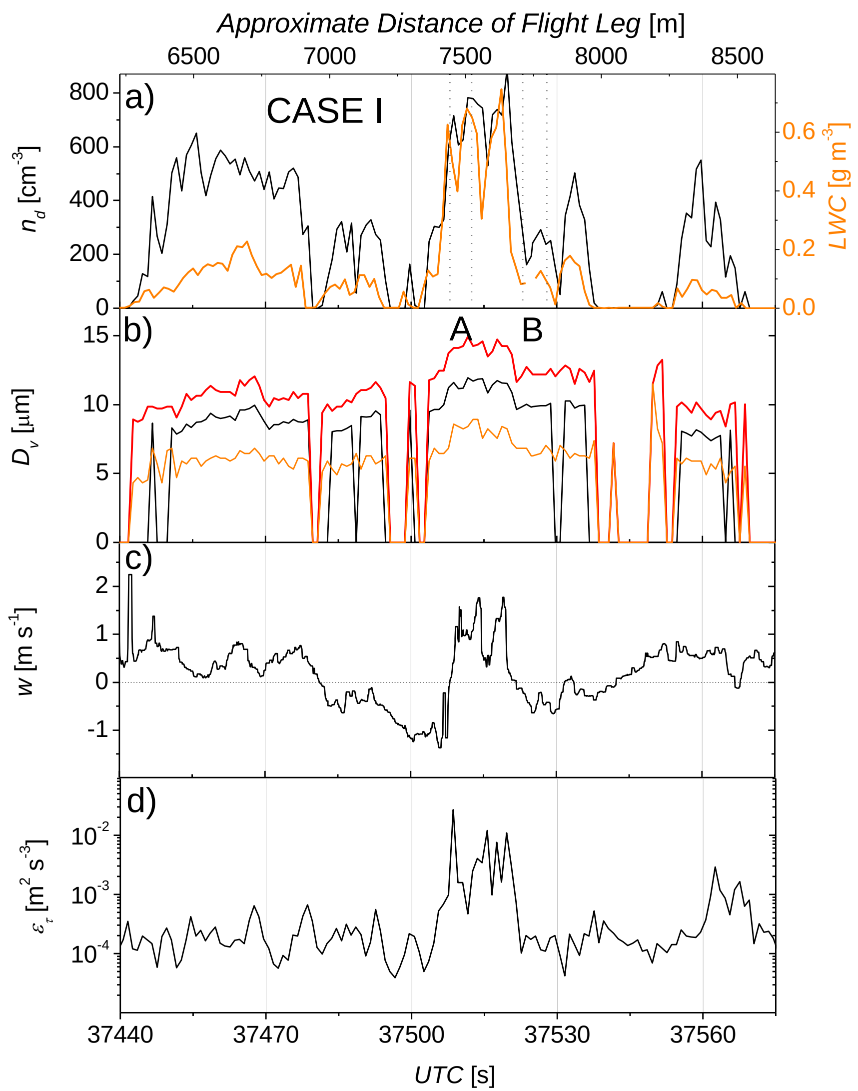
Scale for *Da=1?*

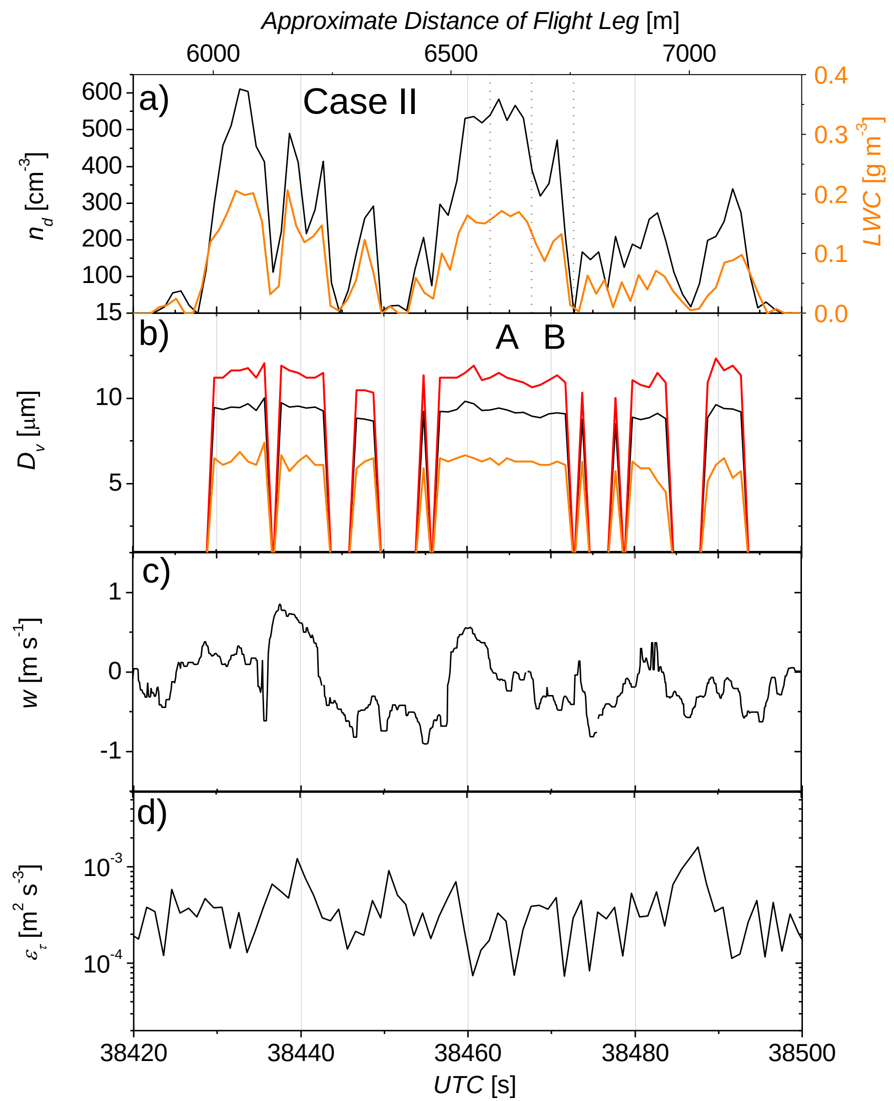
$$l_* = \varepsilon^{1/2}\tau_r^{3/2}$$



Lehmann et al., *J. Atmos. Sci.* (2009)

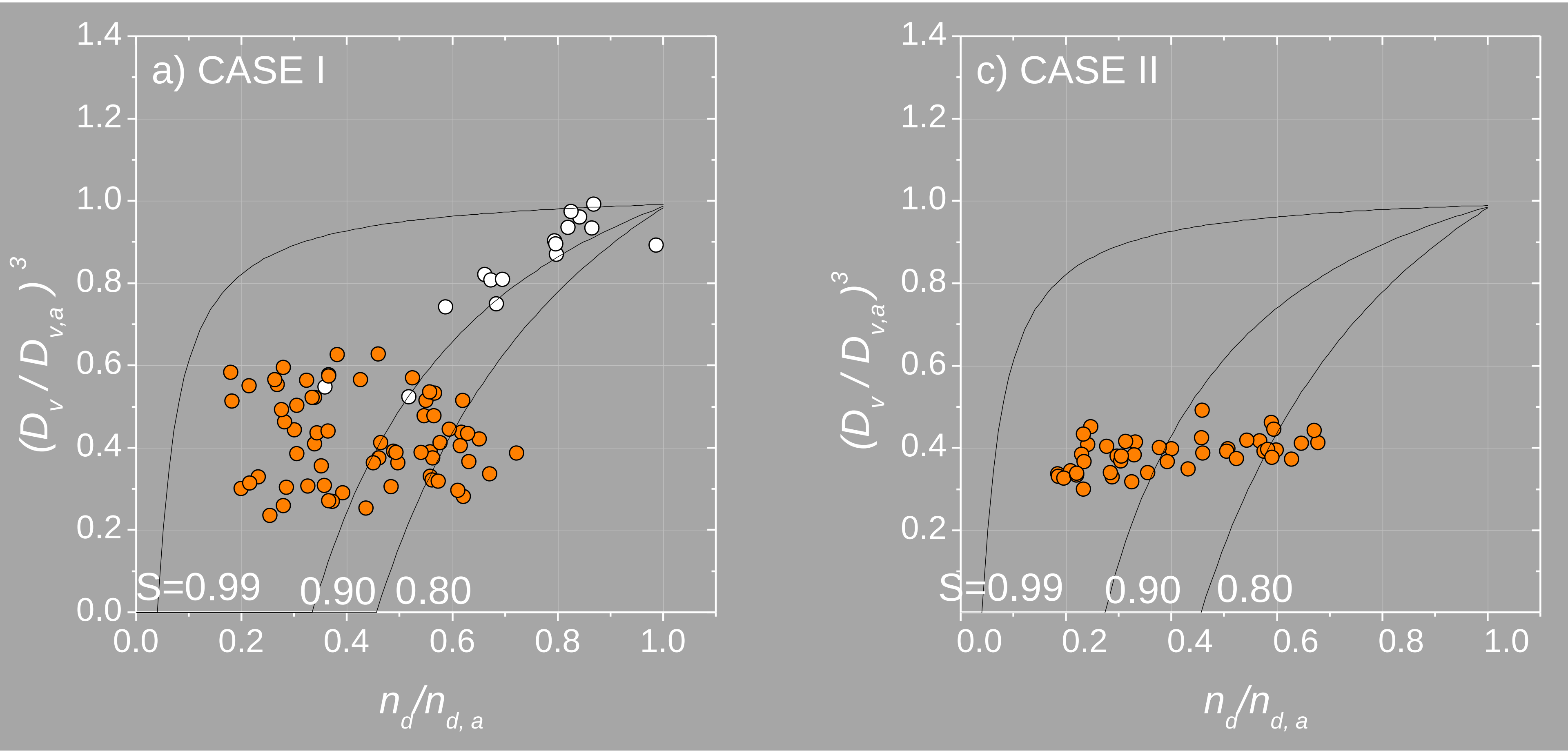
A tale of two clouds…





Lehmann et al., *J. Atmos. Sci.* (2009)

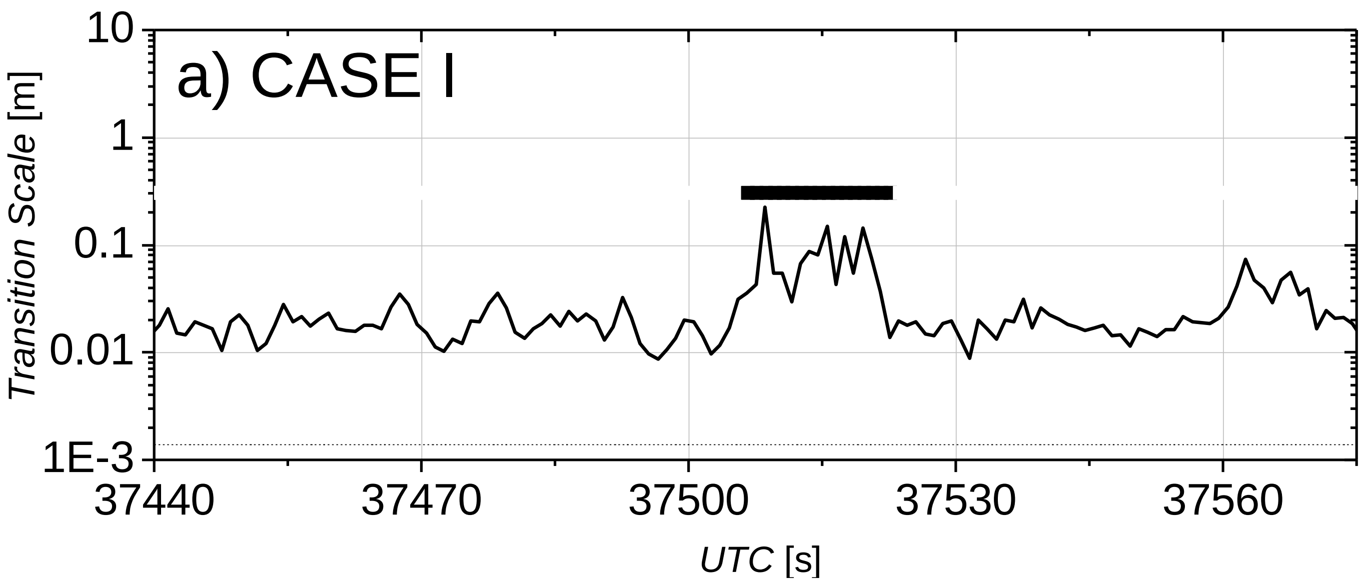
Examples… continued

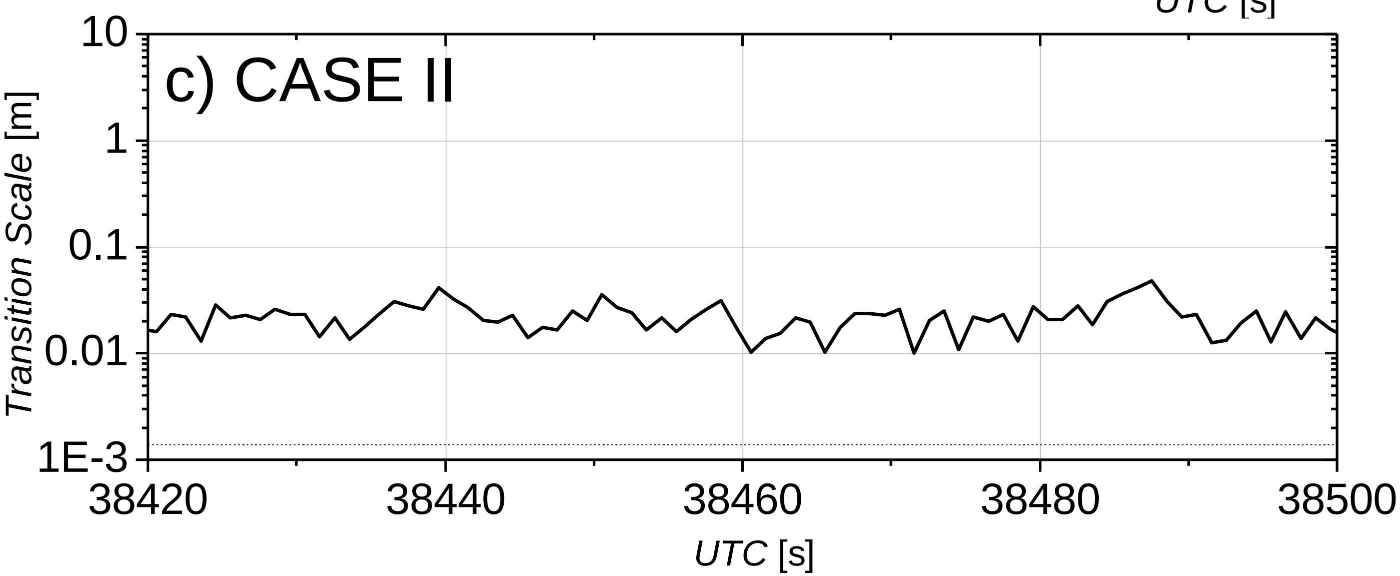


Lehmann et al., *J. Atmos. Sci.* (2009)

Transition length scale…

Homogeneous region

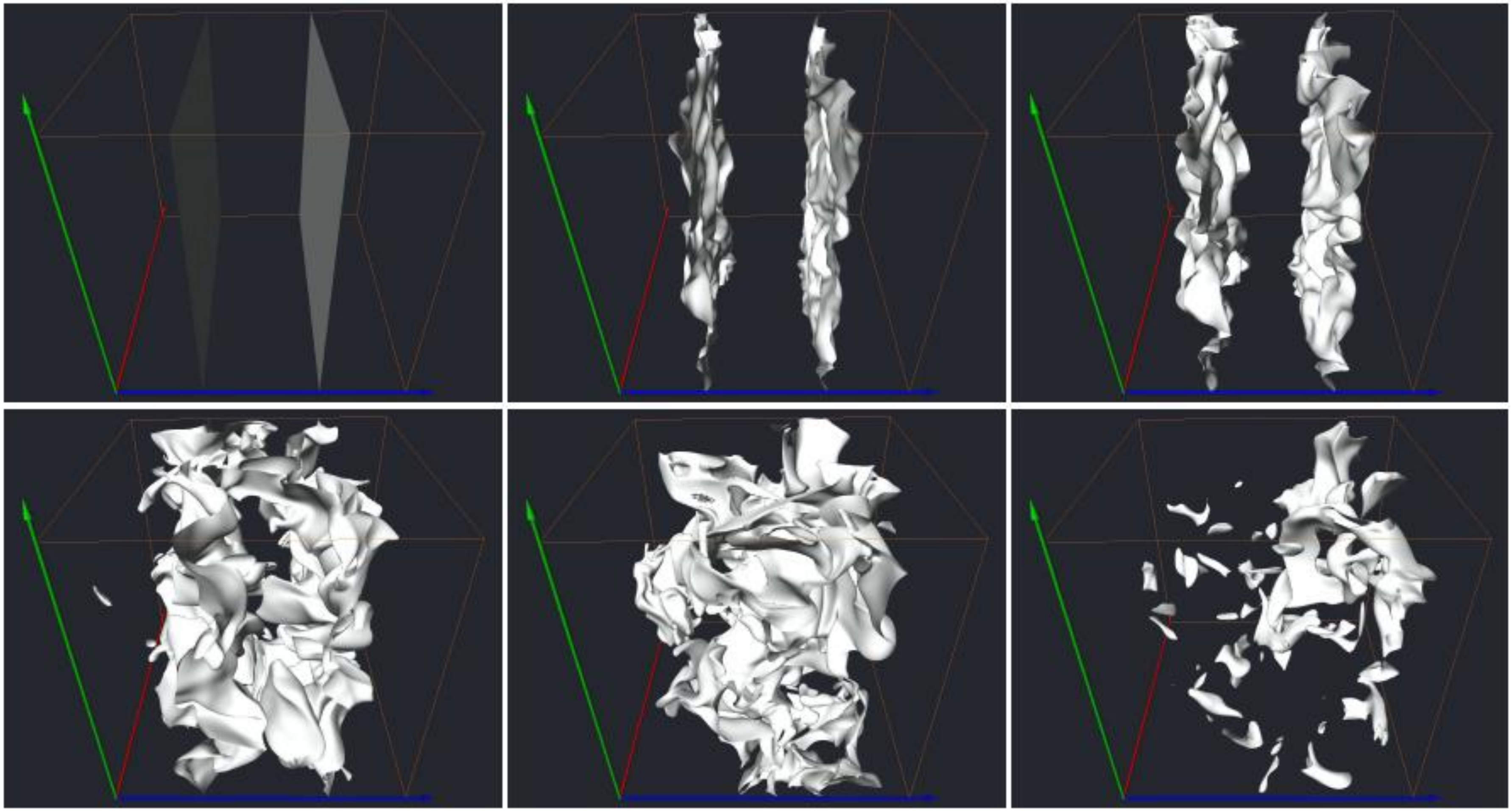




Inhomogeneous

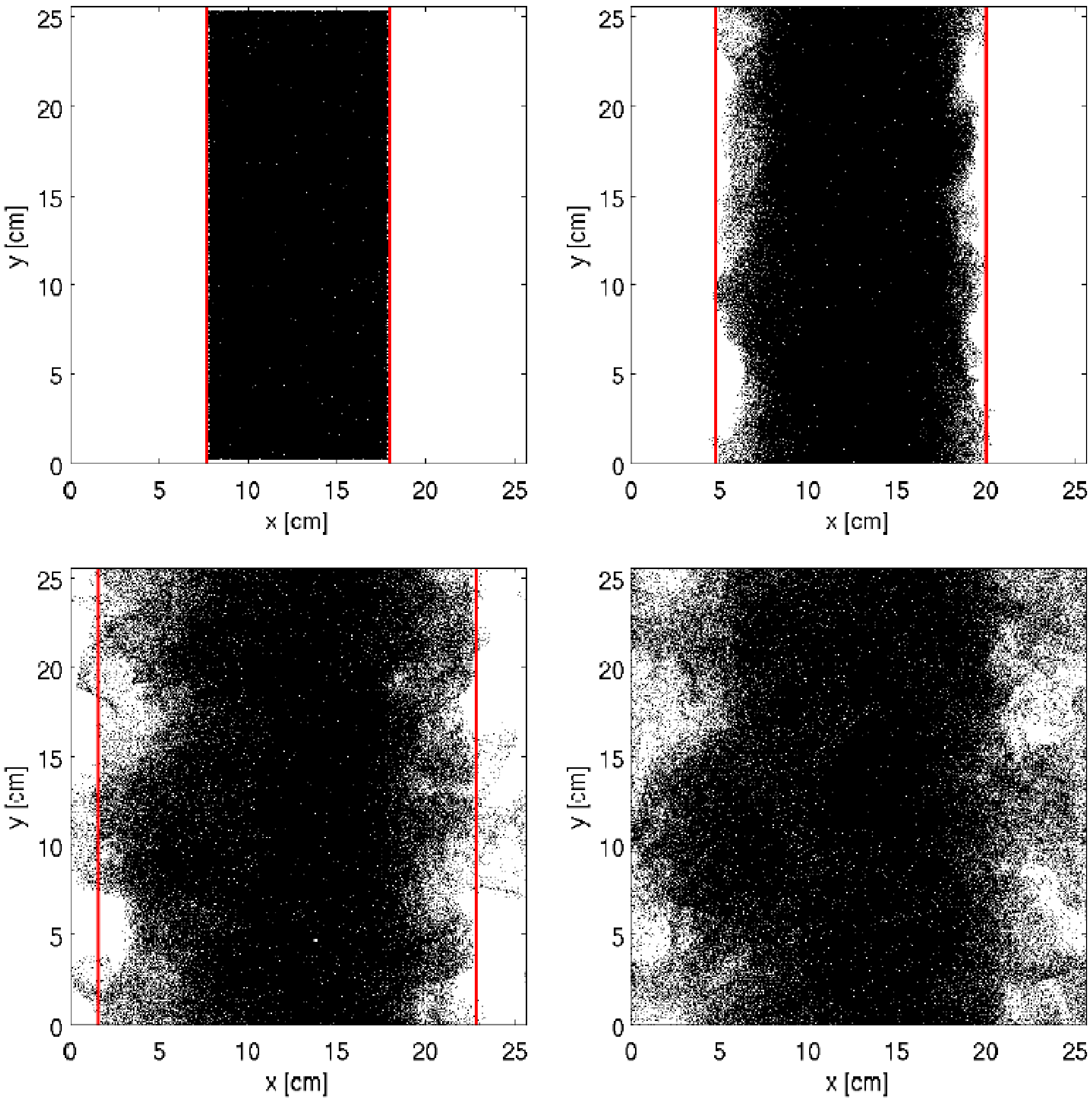
Lehmann et al., *J. Atmos. Sci.* (2009)

DNS of entrainment with Lagrangian droplet dynamics



Kumar, Schumacher, Shaw – MetStroem special issue

Decoupling of vapor afield and droplets…



Kumar, Schumacher, Shaw – MetStroem special issue

Summary

- We have presented several illustrations of how cloud droplets behave in turbulence – both mechanical and thermodynamic coupling with turbulence over a range of scales.
- Inertial particle clustering (in turbulence) has been studied experimentally and via DNS simulation.
- Experiment shows strong *St* scaling. Effects of *Re* & gravity are relatively weak (below experimental uncertainty) for the range covered (R_λ=430--700, Sv~ 0.1--1).
- Experiment and DNS agree well, when proper averaging is performed.
- Drift-diffusion theory developed for *St*<<1 seems to work up to *St*~0.3 for the monodisperse case.
- Inclusion of gravity is important for bidisperse clustering (and collision velocity) - initial check accomplished through addition of electric charge.
- Turbulent mixing of clear and cloudy air leads to a transition from inhomogeneous mixing at large scales, homogeneous mixing at small scales.

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