A Stochastic Closure Approach to Large Eddy Simulation (LES)

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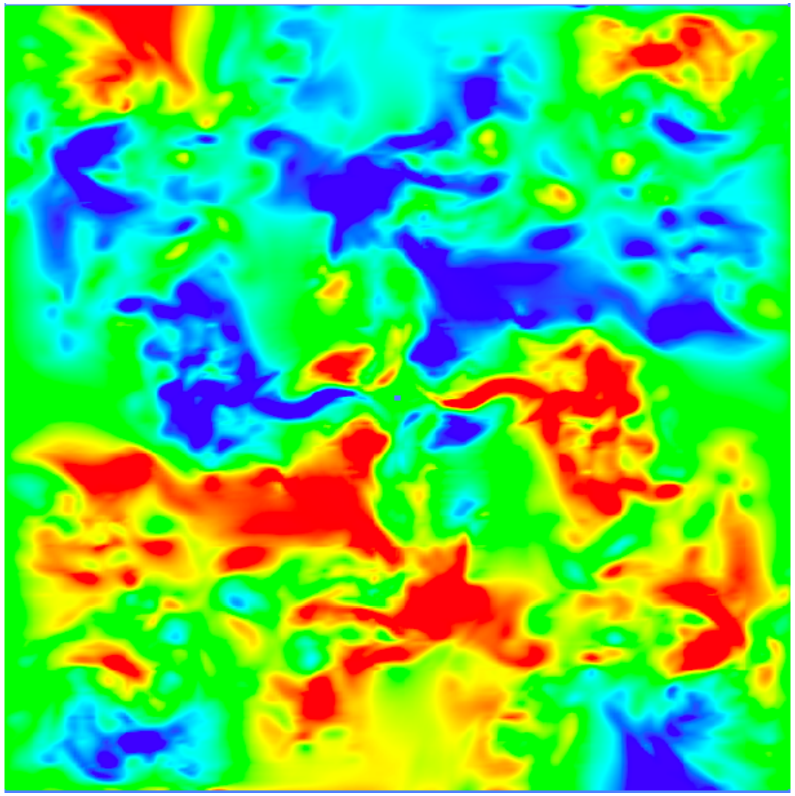
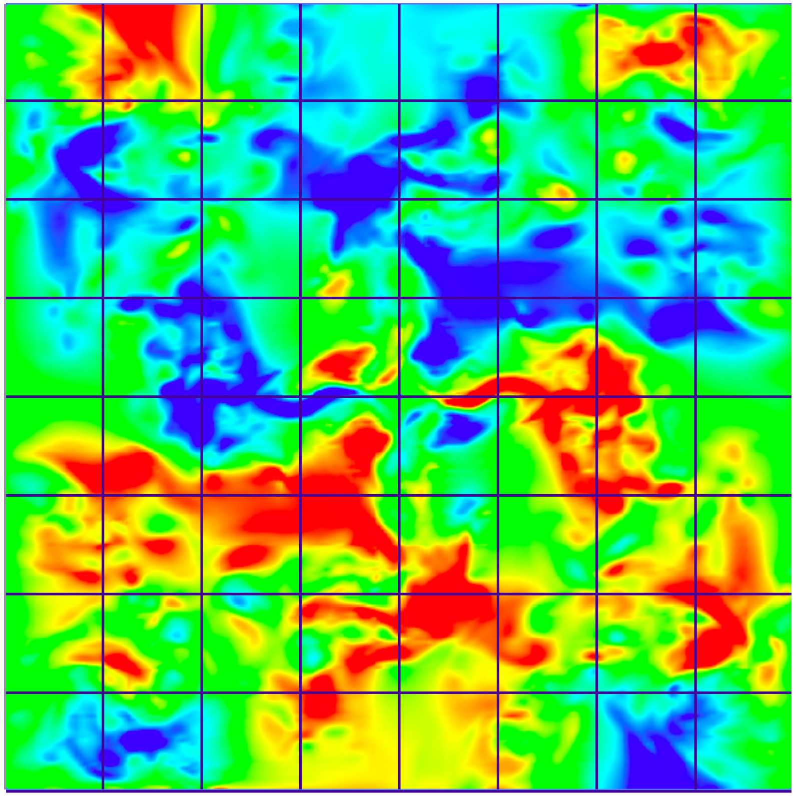
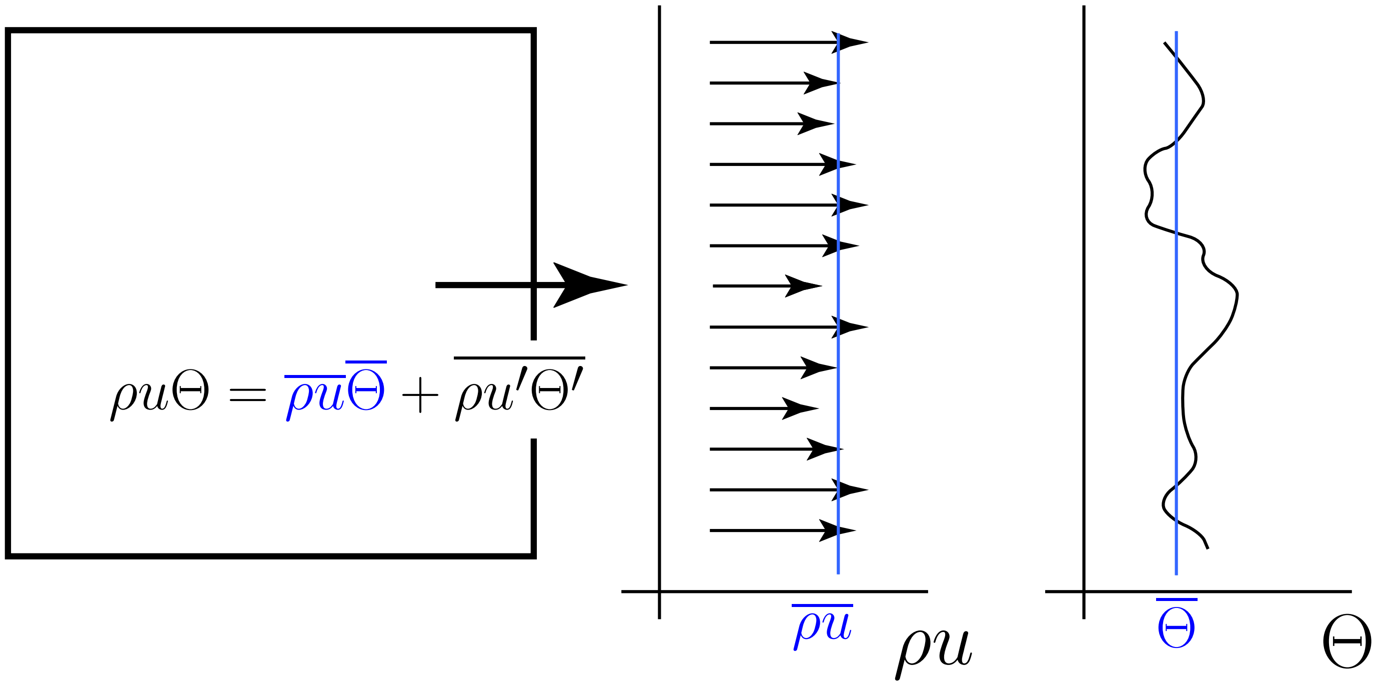




From DNS-Simulations by A. Beck & G. Gassner (see their poster!)

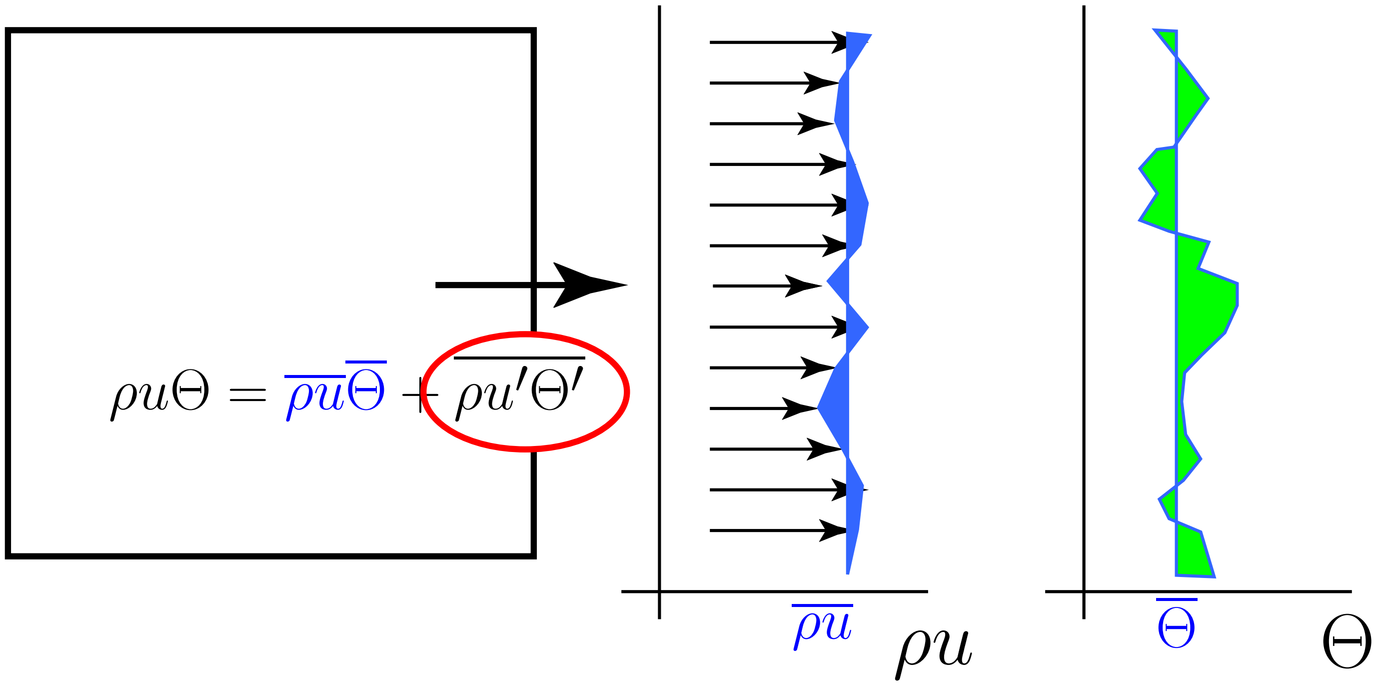
Integral conservation laws

$$\int_\Omega \rho\Theta \, d^3x \, \Big|_{t=t^-}^{t=t^+} = - \int_{t^-}^{t^+} \oint_{\partial\Omega} \rho\Theta \boldsymbol{v} \cdot \boldsymbol{n} \, dA \, dt$$



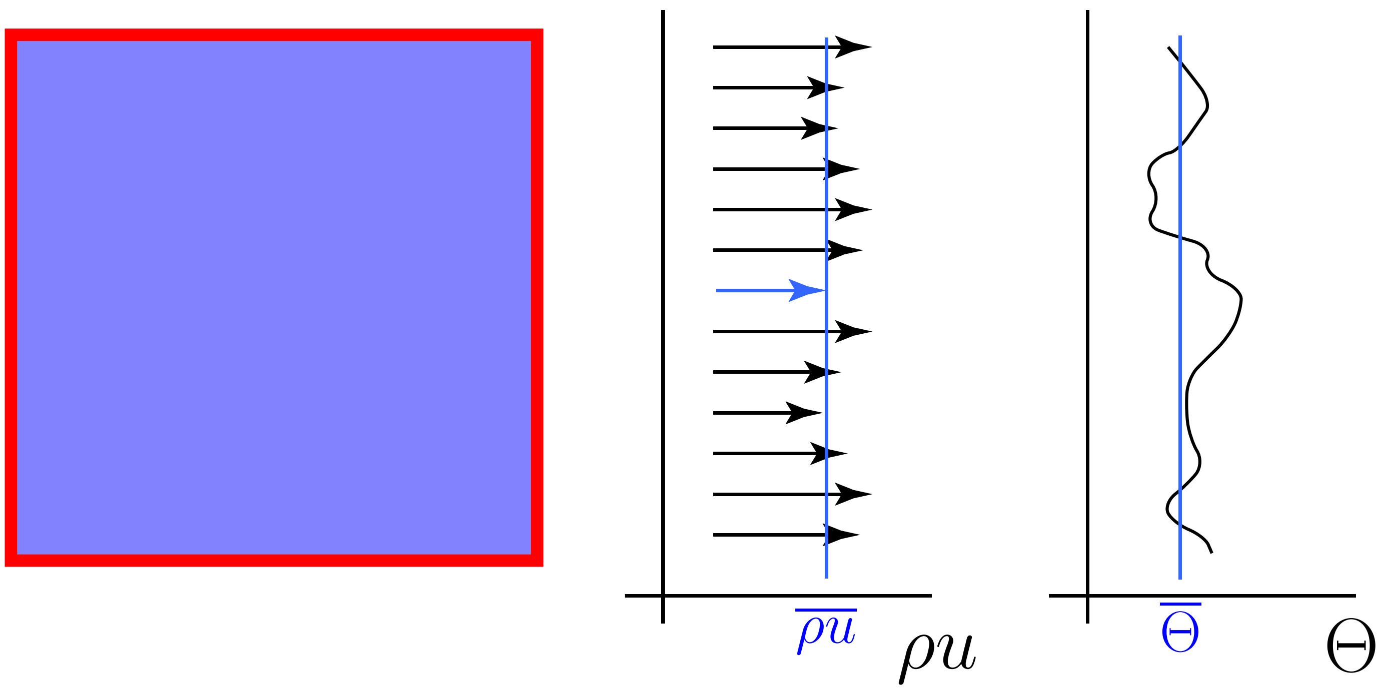
Integral conservation laws

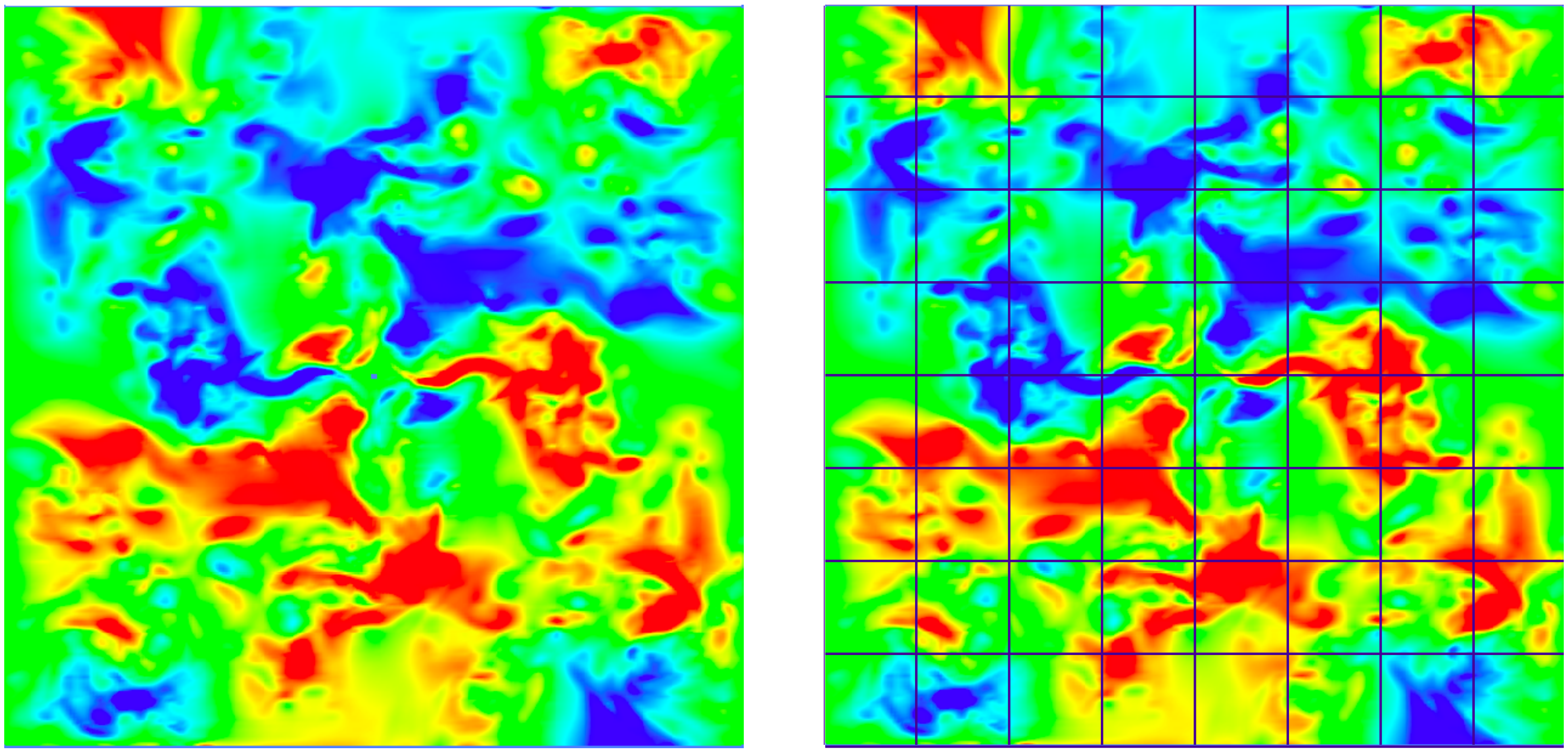
$$\int_\Omega \rho\Theta \, d^3x \Big|_{t=t^-}^{t=t^+} = -\int_{t^-}^{t^+} \oint_{\partial\Omega} \rho\Theta \boldsymbol{v}\cdot\boldsymbol{n} \, dA \, dt$$



Integral conservation laws

$$\overline{\rho\Theta}^{\boldsymbol{C}}\Big|_{t=t^-}^{t=t^+} = \frac{1}{|\Omega|}\sum_j \overline{\rho\Theta\boldsymbol{v}\cdot\boldsymbol{n}}_j^{\boldsymbol{\partial C}} A_j$$





- **finite volumes** + **DG-type**, **stochastic** subgrid scale representation
- stochastic integration of resulting fluxes on coarse-grid interfaces

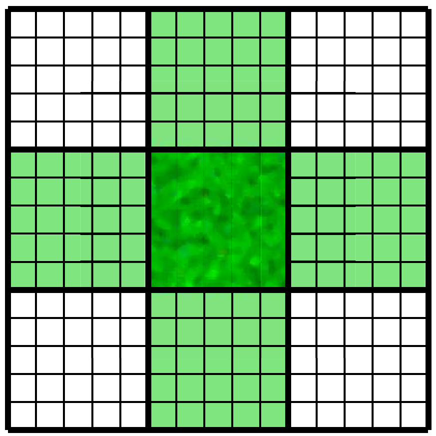
Approach I:

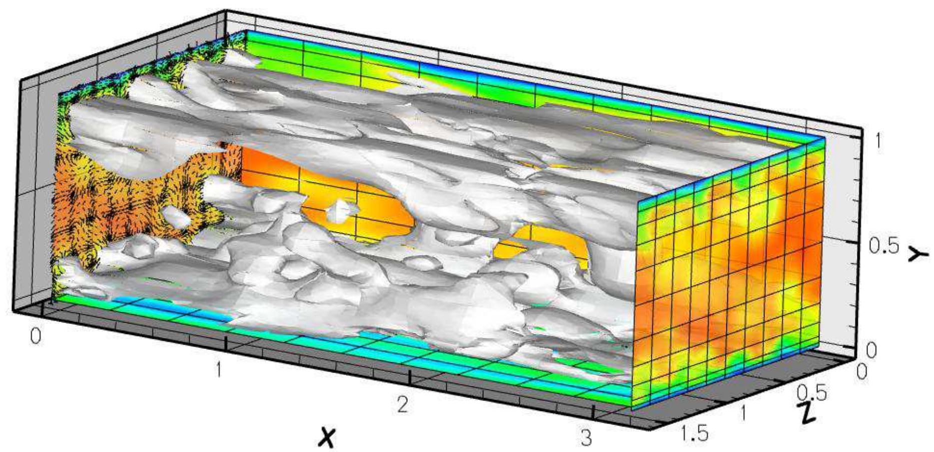
fine-grid space-time patterns ⇔ coarse-grid stencil data

Key Issues:

- **Structure of fluctuations**
- Fluctuations vs. mean flow
- Coarse-grid dynamics

FEM-BV-VARX*–Analysis of DNS data:





DNS by G. Gassner (IAG, Uni Stuttgart)

* I. Horenko, Ph. Metzner, USI, Lugano

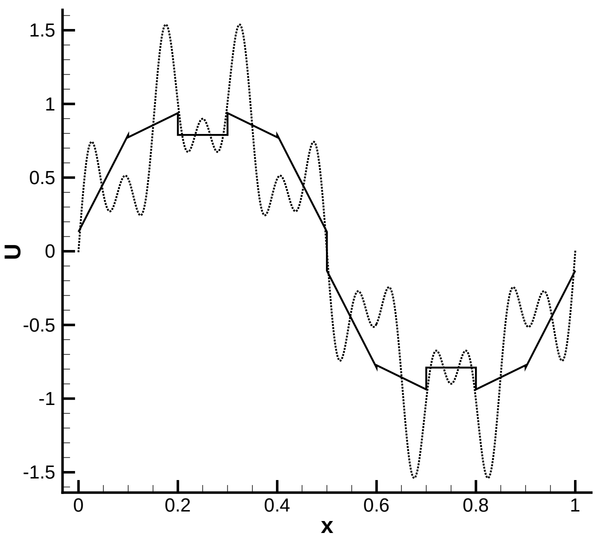
Approach I:

smooth part of flow $\Leftrightarrow$ coarse-grid stencil data

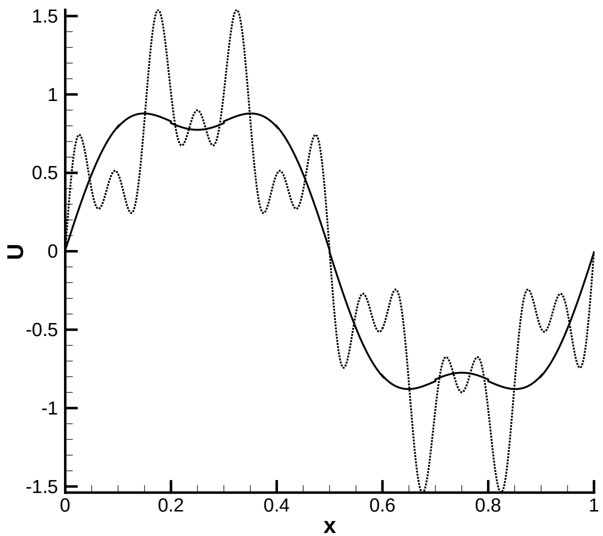
High-order reconstruction* of coarse-grid DNS data:

Key Issues:

- Structure of fluctuations
- **Fluctuations vs. mean flow**
- Coarse-grid dynamics



2nd Order



11th Order

– minimize numerical dissipation

– extract fluctuation effects $\overline{\rho u' \Theta'}^{\partial\Omega}$

*** A. Beck, G. Gassner, C.-D. Munz (IAG, Uni Stuttgart)**

see A. Beck & G. Gassner at their poster!

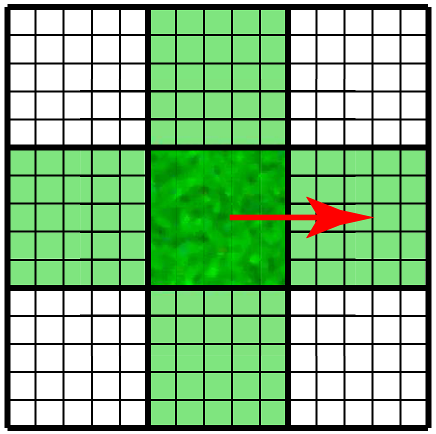
Approach II: (akin to ILES*)

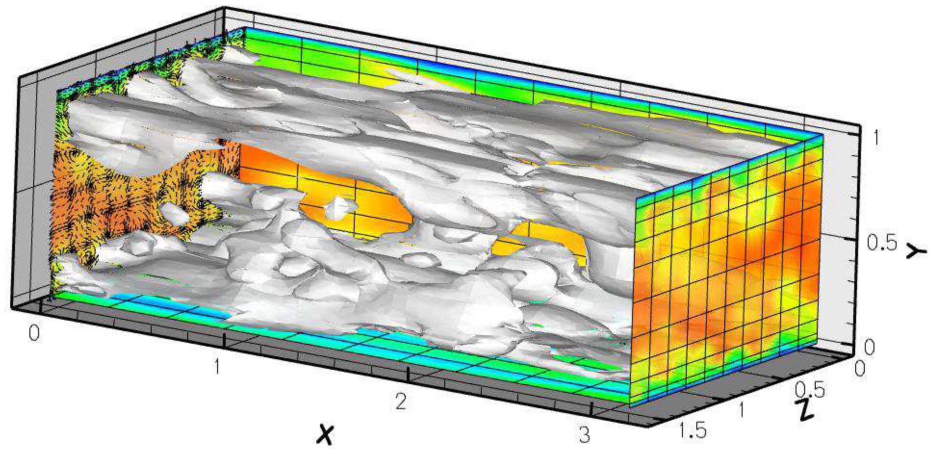
flux fluctuations ⇔ coarse-grid stencil data

Key Issues:

- **Struct. of flux–fluctuations**
- Fluctuations vs. mean flow
- Coarse-grid dynamics

FEM-BV-VARX*–Analysis of DNS data:





DNS by G. Gassner (IAG, Uni Stuttgart)

* **I. Horenko, Ph. Metzner, USI, Lugano**

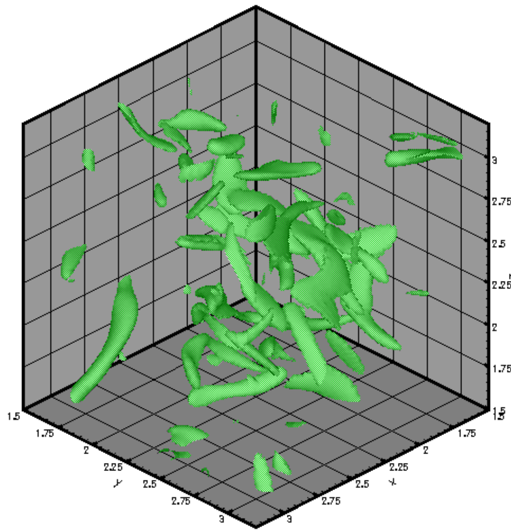
* **see also lectures by N. Adams; S. Remmler & S. Hickel**

Can we reproduce the coarse-grained evolution?

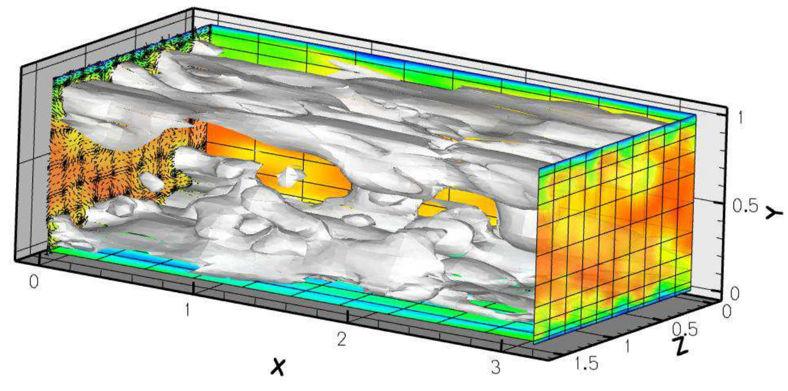
(in a statistical sense,)

Key Issues:

- Structure of fluctuations
- Fluctuations vs. mean flow
- **Coarse-grid dynamics**



M. Uhlmann (CIEMAT, Madrid)



G. Gassner (IAG, Uni Stuttgart)

via

I direct evaluation of conservation law integrals

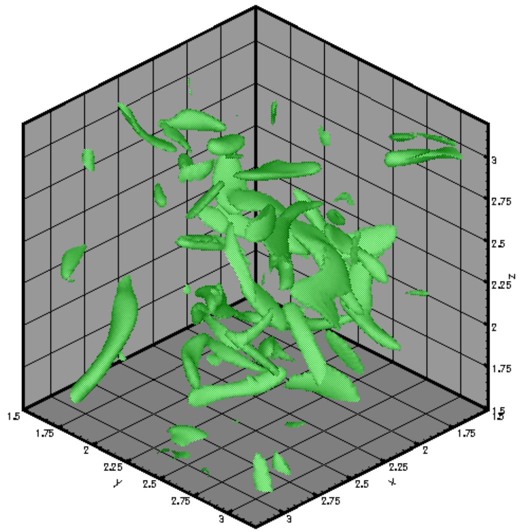
II high-order solver + FEM-BV-Markov-VARX–fluxes

Can we reproduce the coarse-grained evolution?

(in a statistical sense, **or when can we claim success??**)

Key Issues:

- Structure of fluctuations
- Fluctuations vs. mean flow
- **Coarse-grid dynamics**



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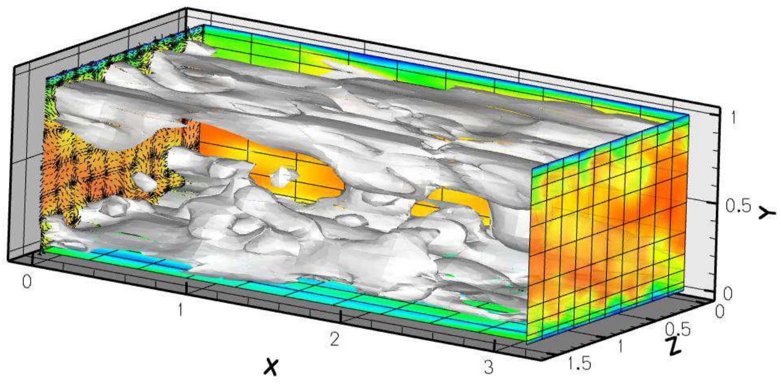

G. Gassner (IAG, Uni Stuttgart)

via

I direct evaluation of conservation law integrals

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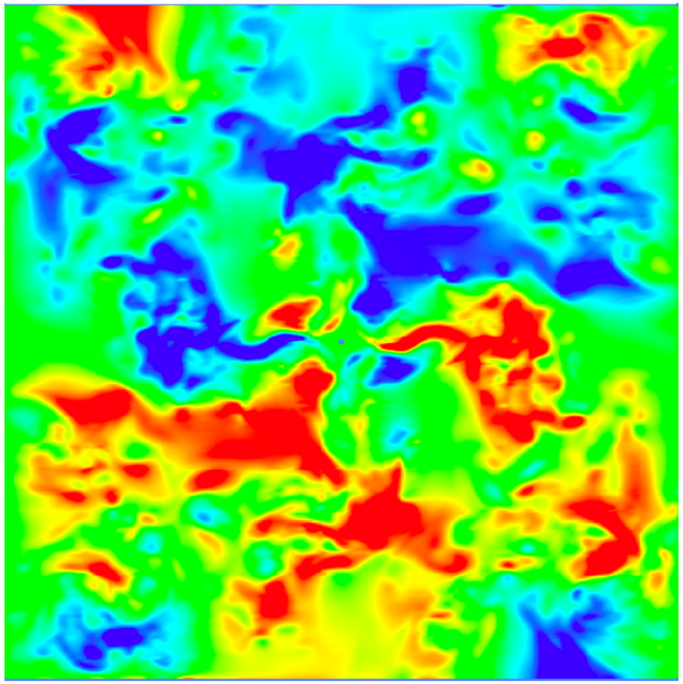
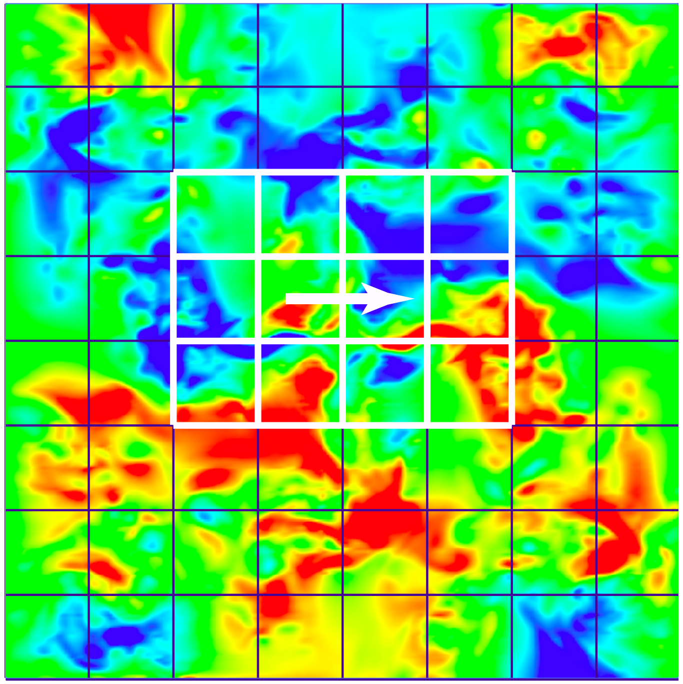
Approach II:

finite volumes + **stencil-conditioned** subgrid scale **flux modelling**





FEM-BV-VARX models

$$\boldsymbol{f}_t = \mu(t) + \mathbf{A}(t)\phi_1(\boldsymbol{f}_{t-\tau}, \ldots, \boldsymbol{f}_{t-m\tau}) + \mathbf{B}(t)\phi_2(\boldsymbol{u}_t) + \epsilon_t.$$

$$\mu(t) = \sum_{i=1}^{K} \gamma_i(t)\mu_i, \quad \mathbf{A}(t) = \sum_{i=1}^{K} \gamma_i(t)\mathbf{A}_i, \quad \mathbf{B}(t) = \sum_{i=1}^{K} \gamma_i(t)\mathbf{B}_i,$$

Test case: Channel Turbulence

with intermittency and upscale energy transport (wall $\longrightarrow$ bulk flow)

fine grid DNS simulation:	600 x 385 x 600	512 time steps
coarse “LES-grid”:	50 x 50 x 50	50 time steps

Simulations by M. Uhlmann (2001–2003

Channel flow test case

ILES-type stochastic coarse grid flux reconstruction via FEM-BV-X

$$\Delta F(t,x) = \sum_{\nu=1}^{K} \gamma_{\nu}^{I}(t,x)\left(\mu_{\nu} + a_{\nu}\Delta F_{\text{1st}} + b_{\nu}\Delta F_{\text{2nd}} + c_{\nu}\Delta F_{\text{WENO}} + d_{\nu}\varepsilon\right)(t,x)$$

for cell interfaces

$$(t,x) = \left(t^{n+1/2}, x_{I}\right) \qquad \text{where} \qquad I = i+\tfrac{1}{2},\ j,\ k \qquad etc.$$

and stencil-based upwind-fluxes

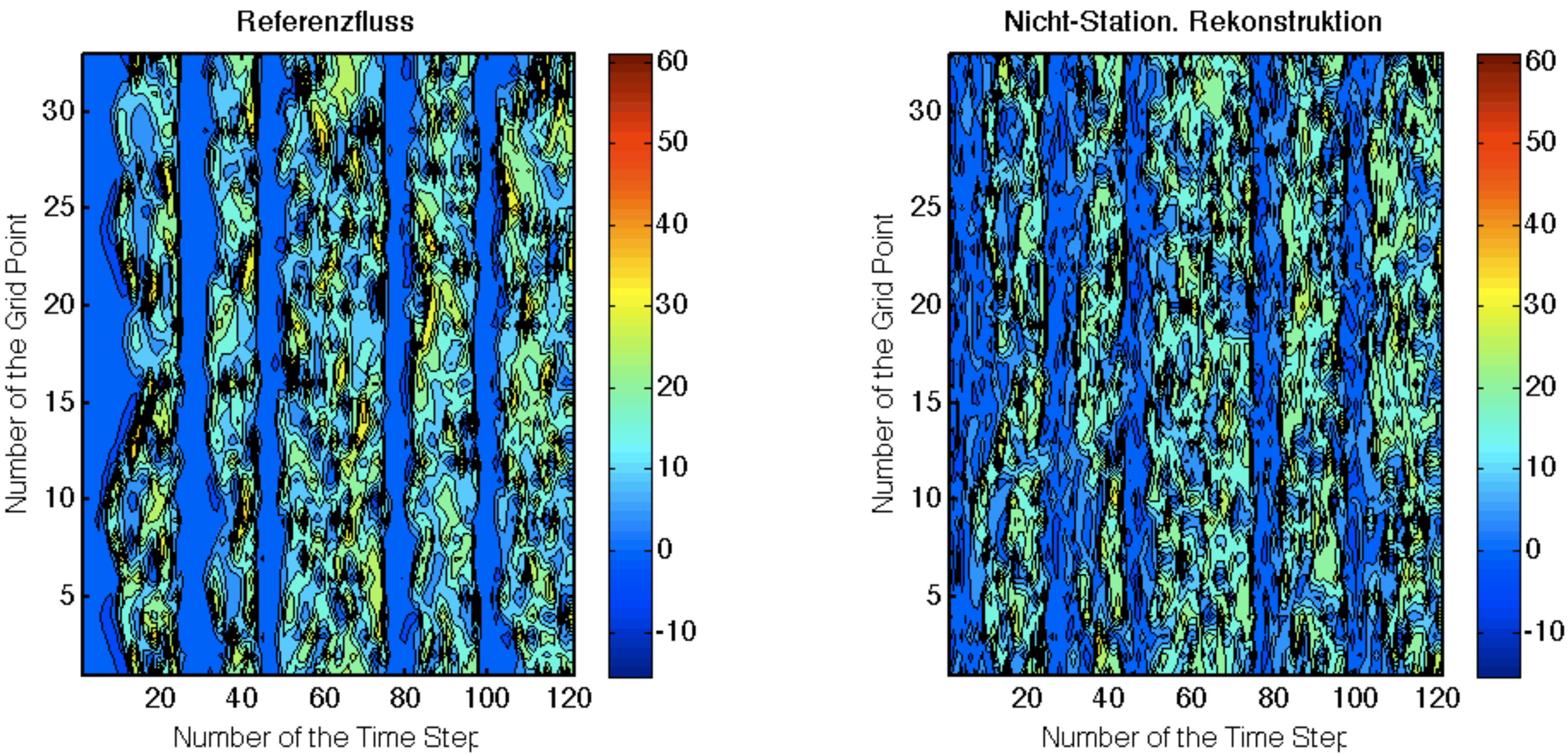
$$\Delta F_{\alpha} = F^{\text{HLLE}}\left(u_{\alpha}^{\text{left}}, u_{\alpha}^{\text{right}}\right) - F_{\text{central}}\,, \qquad \alpha \in \{\text{1st}, \text{2nd}, \text{WENO}\}$$

Channel flow test case

First results with:

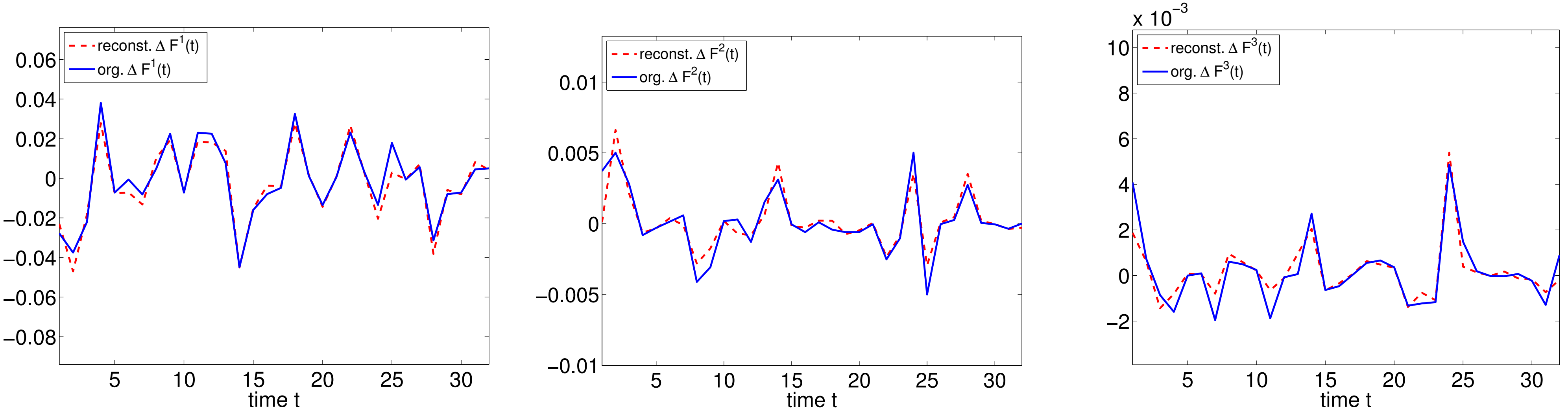
- $\gamma_\nu^{\boldsymbol{I}}$ still data-based
- No. of regimes $K = 3$; No. of regime transitions $\left\|\gamma_\nu^{\boldsymbol{I}}\right\|_{\mathrm{BV}} = 6$ **fixed**

What we qualitatively would have expected:

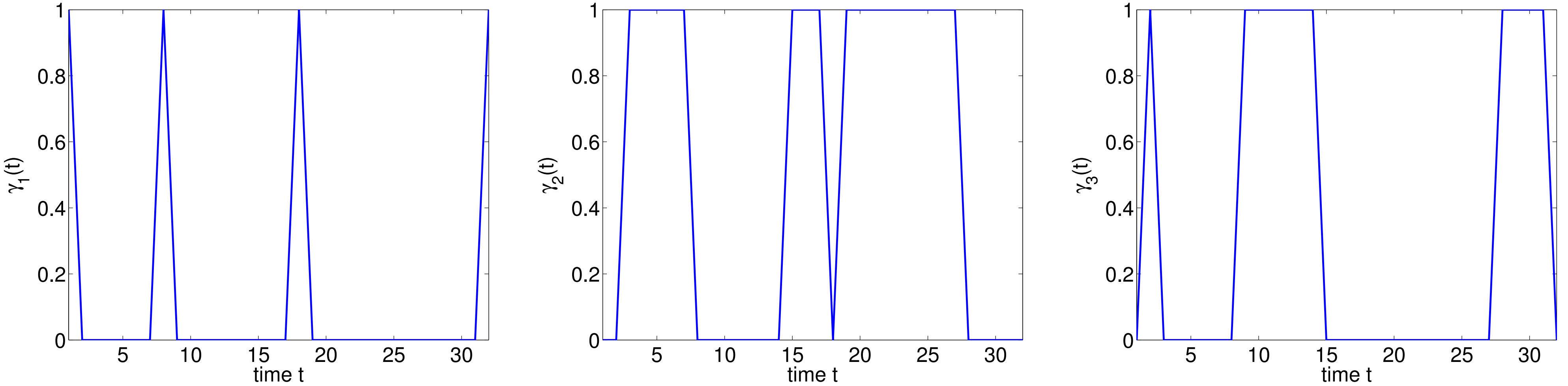


y-directed cell face near channel center

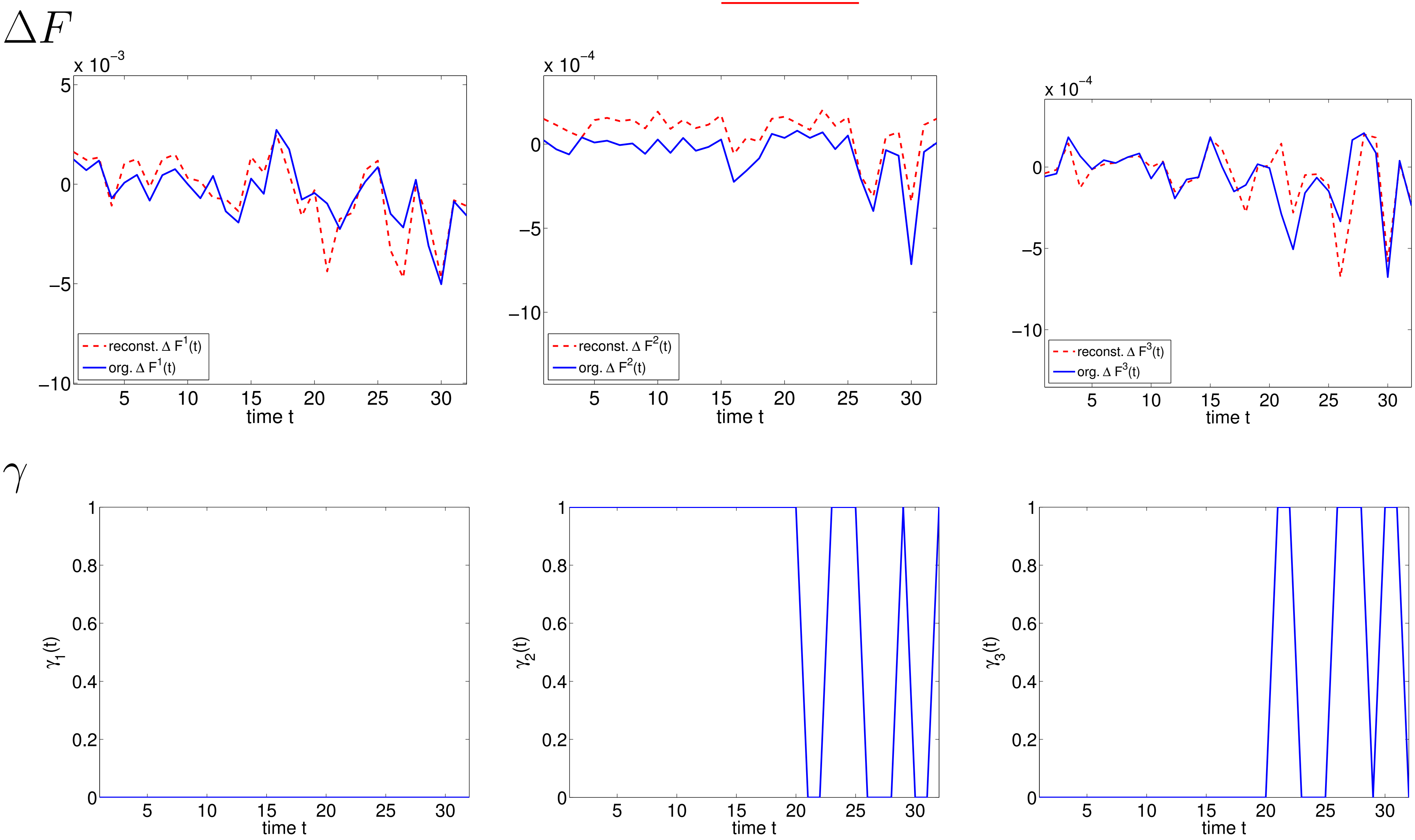
ΔF



γ

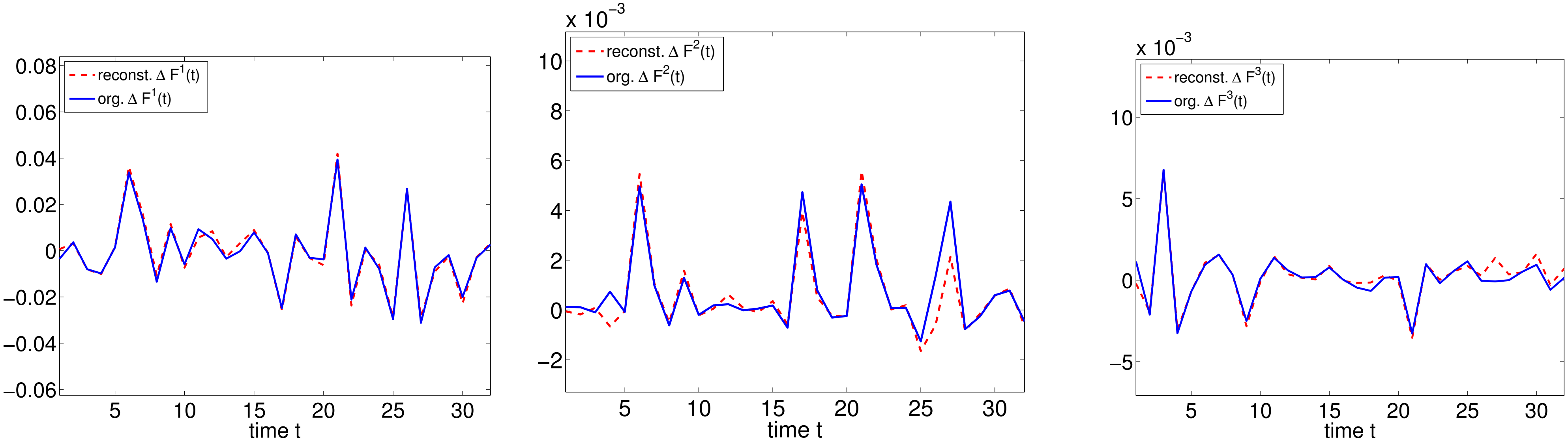


y-directed cell face one cell off the wall

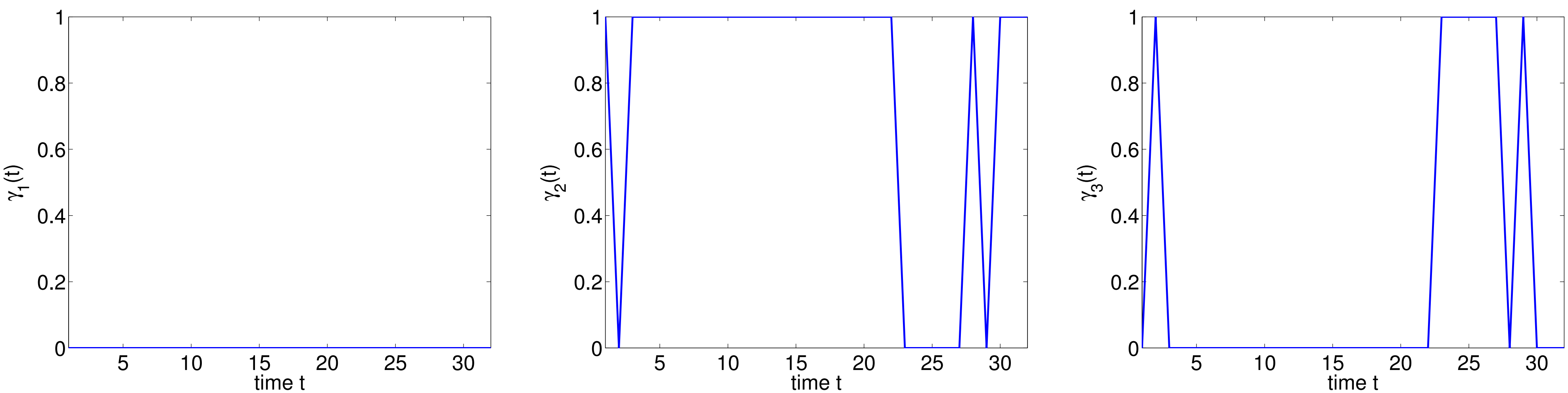


y-directed cell face four cells off the wall

ΔF



γ



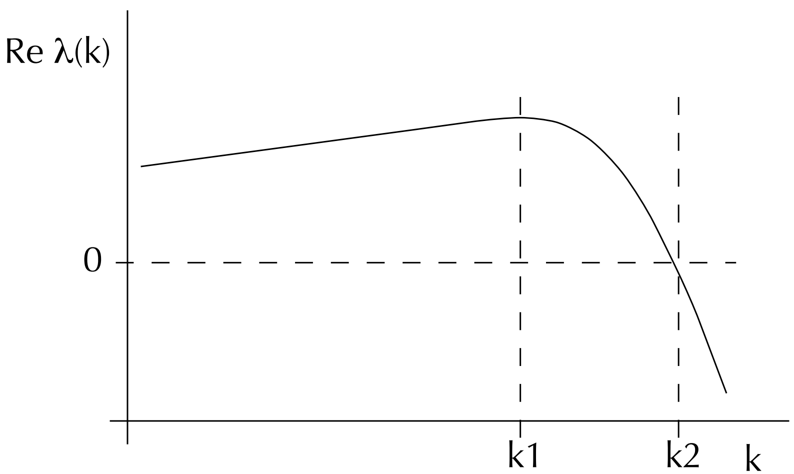
Further steps

- trial coarse-grid run with reconstructed fluxes
- selfconsistent closure with (discrete) stochastic process for γ_{ν}^{I}
- back to Approach I
- Selfconsistent multi-grid-based parameter learning

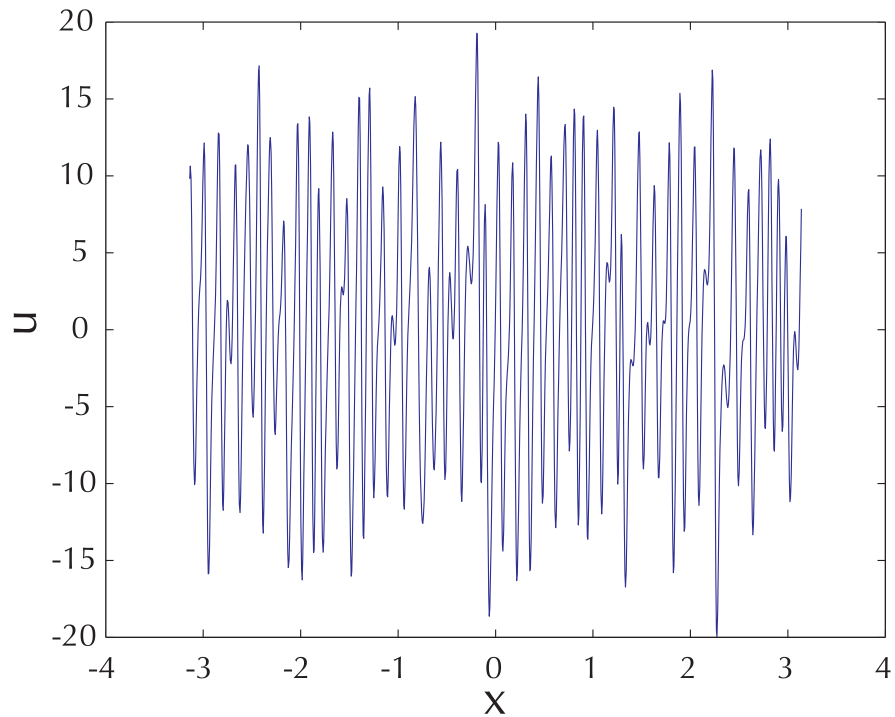
1D Test Problem

Kuramoto-Sivashinsky Equation

$$\frac{\partial u}{\partial t} + \frac{\partial}{\partial x}\left(\frac{u^2}{2} + \boldsymbol{\varepsilon} u_x + \boldsymbol{\kappa} u_{xxx}\right) = 0$$



chaotic sample solution: $\boldsymbol{\varepsilon} = 0.1\,, \quad \boldsymbol{\kappa} = 2 \cdot 10^{-5}$



1D Test Problem

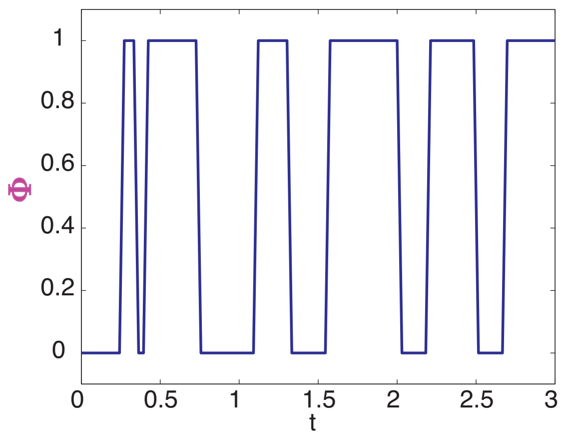
Kuramoto-Sivashinsky + Excited Burgers

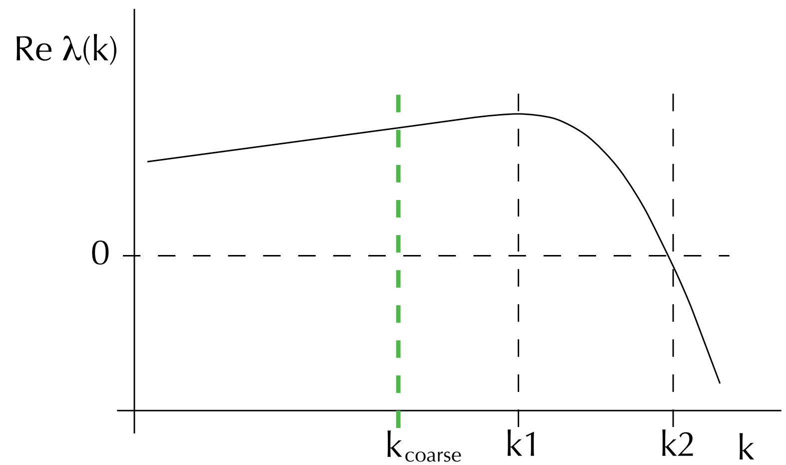
$$\frac{\partial u}{\partial t} + \frac{\partial}{\partial x}\left(\frac{u^2}{2}\right) = \boldsymbol{\Phi}(t)\,(-\boldsymbol{\varepsilon} u_{xx} - \boldsymbol{\kappa} u_{xxxx}) + (1 - \boldsymbol{\Phi}(t))\,Q(t,x)$$

long-wave excitation

$$Q(t,x) = \sum_{j=1}^{5} A_j \,\sin(\omega_j\, t)\,\sin(k_j\, x)$$

regime switching





1D Test Problem

ILES-type (stochastic) coarse grid flux reconstruction

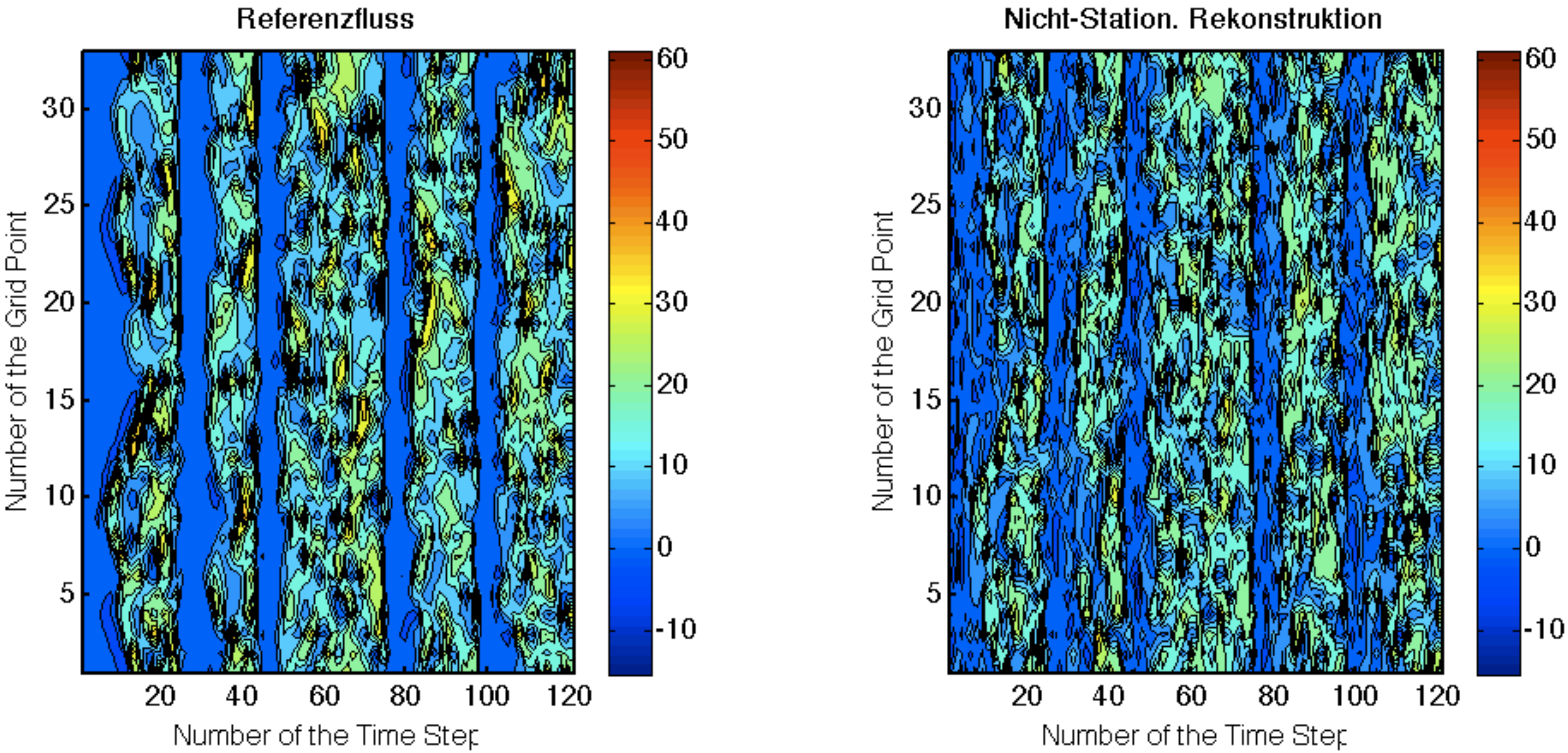
$$f_{KS}(t,x) = \sum_{i=1}^{K} \gamma_i(t,x) \left(\mu_i + a_i f_{1\text{st}} + b_i f_{2\text{nd}} + c_i f_{\text{WENO}} + d_i \varepsilon \right)(t,x)$$

for cell interfaces

$$(t,x) = \left(t^{n+1/2}, x_{j+1/2} \right)$$

and stencil-based **Burgers**-fluxes

$$f_\alpha = f_{\text{Burg}}^{\text{HLLE}} \left(u_\alpha^{\text{left}}, u_\alpha^{\text{right}} \right) , \qquad \alpha \in \{1\text{st}, 2\text{nd}, \text{WENO}\}$$



$$\|F - F_{rec}\|_2 = 0.14\,\|F\|_2$$