Spectral Structure of Stratified Turbulence
Predictions by Implicit LES

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Outline

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Introduction: Stratified Turbulence

Stratified turbulence

- Turbulence in density stratified media (ocean, atmosphere)
- Small scale turbulence is the basic mechanism of energy dissipation in geophysical flows

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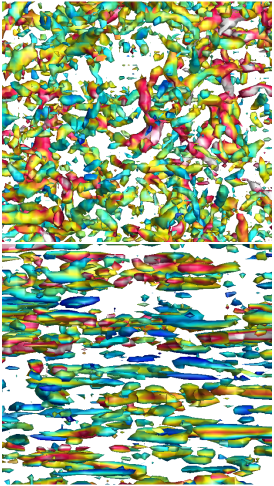
- Traditional turbulence parametrizations involve assumptions on turbulence isotropy
- Stratification damps vertical motion
  $\Rightarrow$ turbulence is strongly anisotropic



Introduction: Governing Equations

Non-dimensional Boussinesq equations:

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\begin{aligned}
\nabla \cdot \mathbf{u} &= 0 \\
\partial_t \mathbf{u} + \nabla \cdot (\mathbf{u}\mathbf{u}) &= -\nabla p - \frac{\rho}{\mathrm{Fr}^2}\mathbf{e}_z + \frac{1}{\mathrm{Re}}\nabla^2 \mathbf{u} + \mathbf{F} \\
\partial_t \rho + \nabla \cdot (\rho \mathbf{u}) &= -\mathbf{u} \cdot \mathbf{e}_z + \frac{1}{\mathrm{Pr}\,\mathrm{Re}}\nabla^2 \rho
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Parameters:

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\mathrm{Fr} &= \frac{\mathcal{U}}{N\mathcal{L}} \\
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We set $\mathrm{Pr} = 0.7$ and control the flow regime by varying Fr and Re.

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Conditions for stratified turbulence:

$0 < \mathrm{Fr} \ll 1 \qquad \mathrm{Re} \gg 1 \qquad \mathrm{Re}\mathrm{Fr}^2 \gg 1$

Introduction: Energy Transport

Transport equation for instantaneous turbulence kinetic energy $E_k = \frac{1}{2} u_i u_i$ (zero mean velocity):

$$\partial_t E_k + \underbrace{\nabla \cdot (\mathbf{u} E_k)}_{\mathcal{T}} = \underbrace{-\mathbf{u} \cdot \nabla p}_{\mathcal{P}} \underbrace{- \frac{\rho}{\mathrm{Fr}^2} \mathbf{u} \cdot \mathbf{e}_z}_{\mathcal{B}} + \underbrace{\frac{1}{\mathrm{Re}} \left[\nabla^2 E_k + (\nabla \cdot \mathbf{u})^2 \right]}_{\mathcal{D}} + \underbrace{\mathbf{F}}_{\mathcal{F}}$$

- $\mathcal{T}$: turbulent diffusion (advection by turbulent velocity)
- $\mathcal{P}$: pressure transport
- $\mathcal{B}$: buoyancy transport
- $\mathcal{D}$: molecular dissipation and diffusion
- $\mathcal{F}$: forcing

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Implicit LES

Discretization of LES equations leads to a modified conservation law:

$$\partial_t \overline{u}_N + \partial_x \overline{F(\overline{u})} + \cdots = \tilde{\mathcal{G}}_{SGS} + \mathcal{G}_{num}$$

Interference between subgrid-scale (SGS) stresses and grid truncation error

- $\rightarrow$ numerical instability
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→ no grid convergence

Solution

combine discretization scheme and SGS model

⇒ **implicit LES** (ILES)

one possible realization: **ALDM** (Adaptive Local Deconvolution Method)

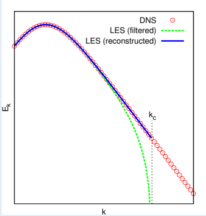
Hickel *et al.*, JCP 213 (2006) and PF 19 (2007)

Adaptive Local Deconvolution Method (ALDM)

Reconstruction (soft deconvolution)

- combination of different interpolation stencils according to a measure of smoothness of the solution
- includes four free model parameters
- recovers part of the SGS energy transfer
- improves prediction of anisotropy

$$\tilde{u}_N \approx G_N^{-1} * u_N$$



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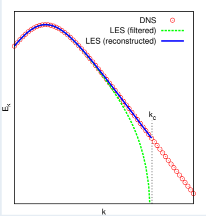

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Regularization

- use a consistent numerical flux function
- one more independent parameter

$$\tilde{F}_{j+1/2} = \frac{1}{4}\left(\tilde{u}^R_{j+1/2} + \tilde{u}^L_{j+1/2}\right)^2 + \sigma_j \left|u_j - u_{j-1}\right| \left(\tilde{u}^R_{j+1/2} + \tilde{u}^L_{j+1/2}\right)$$

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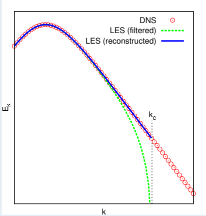

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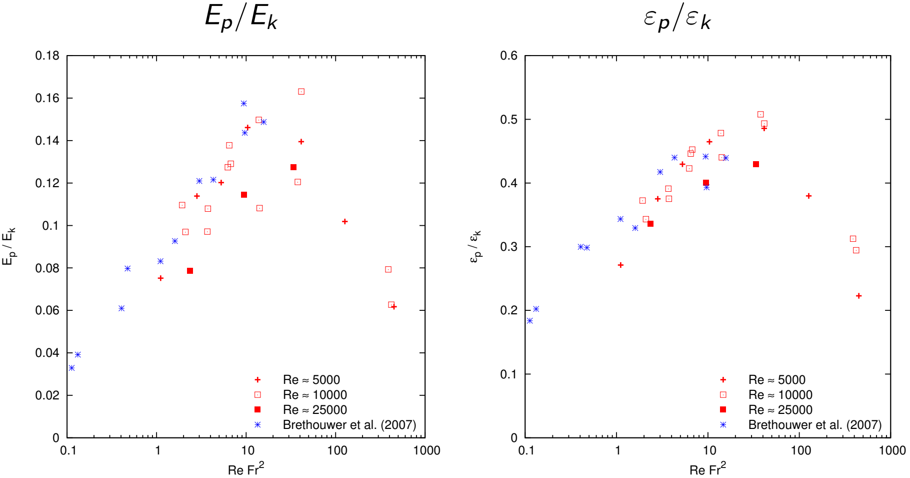
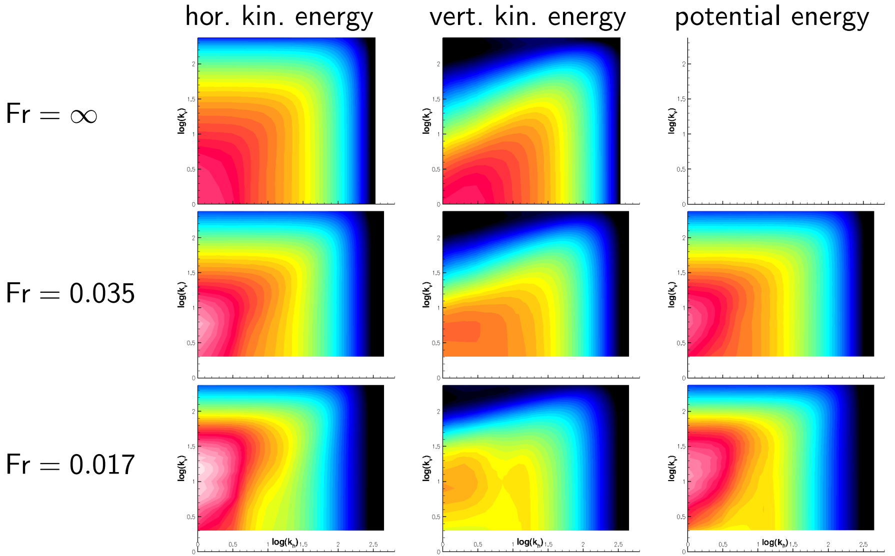
DNS: Simulation Set-up

- homogeneous turbulence in a triple-periodic box
- random horizontal volume force on large horizontal wavenumbers
- spectra averaged in time and horizontal direction
- Reynolds number: $5000 \leq \mathrm{Re} \leq 25\,000$
- Froude number: $0.017 \leq \mathrm{Fr} \leq 0.2$ (+ neutrally stratified case)
- grid size: up to 960^3 grid cells ($\mathrm{Re} = 25\,000$)

DNS: Energy Ratio and Mixing Efficiency



DNS: 2d Energy Spectra



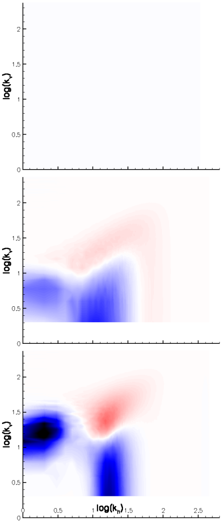
DNS: Energy Transfer Spectra I

$\mathcal{B}(E_v)$

$\mathrm{Fr} = \infty$

$\mathrm{Fr} = 0.035$

$\mathrm{Fr} = 0.017$



DNS: Energy Transfer Spectra I

$\mathcal{B}(E_v)$ $\mathcal{P}(E_v)$

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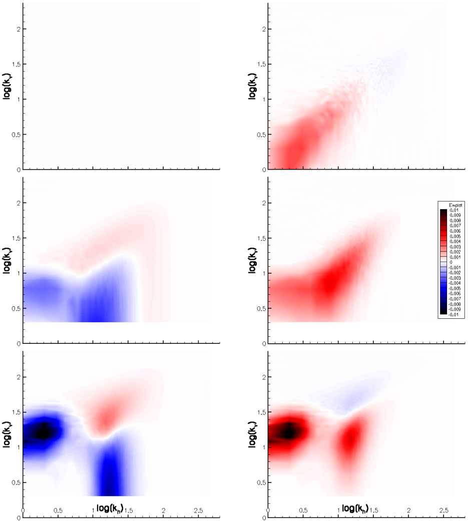


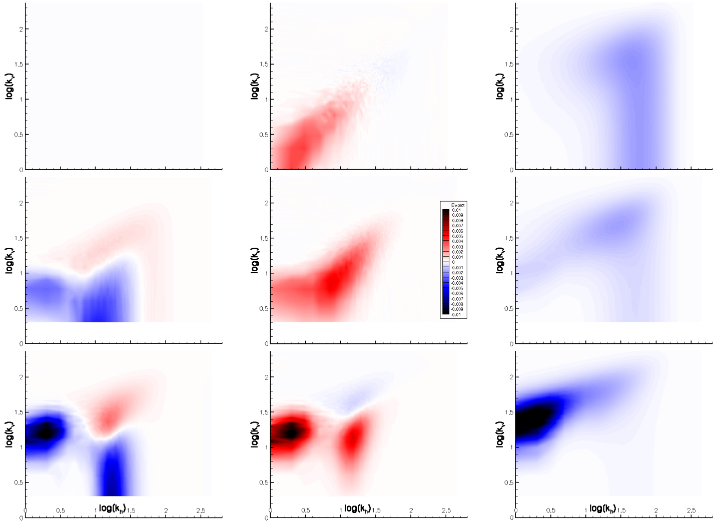
DNS: Energy Transfer Spectra I

$\mathcal{B}(E_v)$ | $\mathcal{P}(E_v)$ | $\mathcal{D}(E_h)$

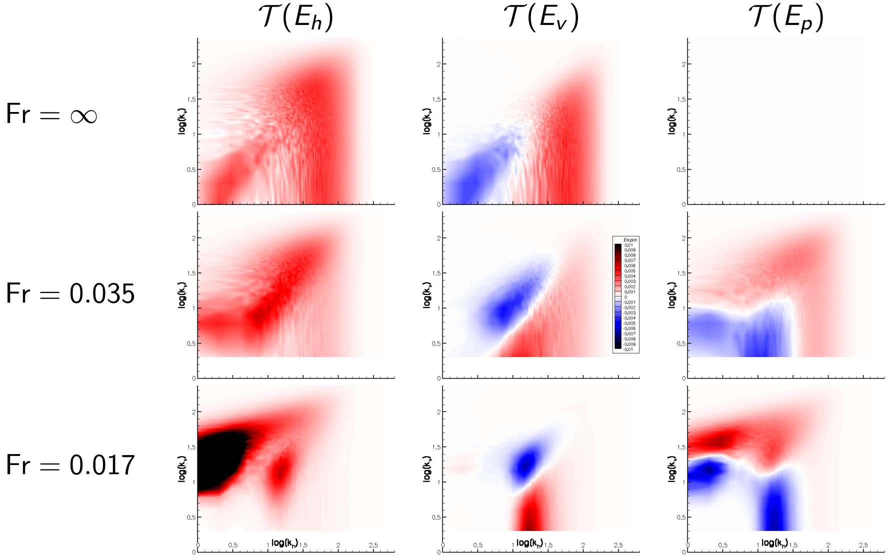
Fr $= \infty$

Fr $= 0.035$

Fr $= 0.017$



DNS: Energy Transfer Spectra II



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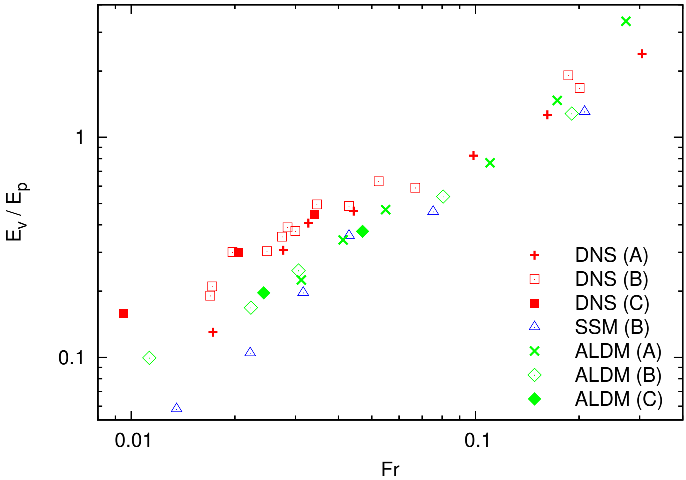
ILES: Potential to Vertical Energy Ratio



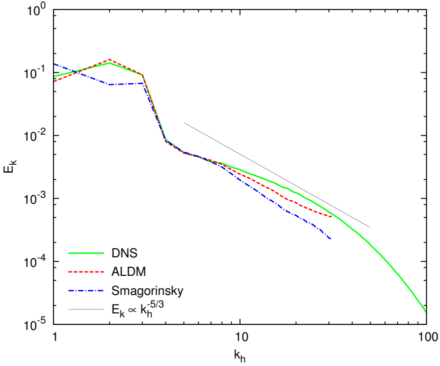
A: $Re \approx 5000$

B: $Re \approx 10000$

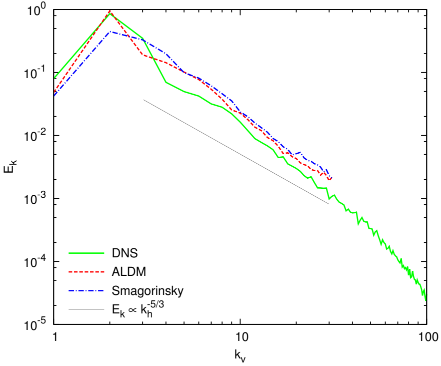
C: $Re \approx 25000$

ILES: Kinetic Energy Spectra I

Weakly stratified case ($Fr = 0.067$, $Re\,Fr^2 = 41.4$)



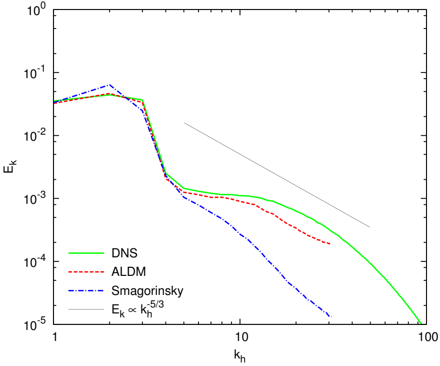
horizontal spectra

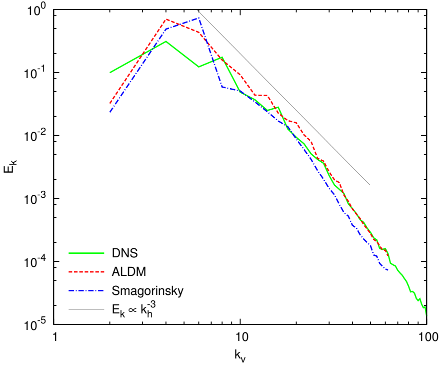


vertical spectra

ILES: Kinetic Energy Spectra II

Strongly stratified case ($\mathrm{Fr} = 0.027$, $\mathrm{Re}\,\mathrm{Fr}^2 = 6.25$)





horizontal spectra

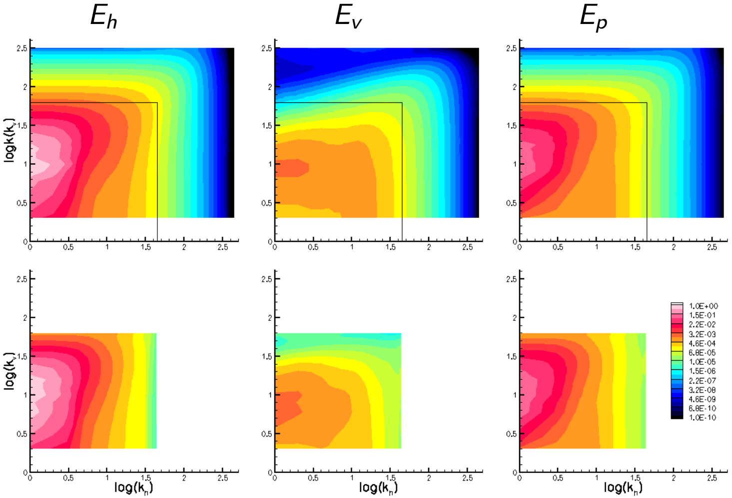
vertical spectra

ILES: 2d Energy Spectra

Strongly stratified case ($\mathrm{Fr} = 0.024$, $\mathrm{Re}\,\mathrm{Fr}^2 = 3.74$)

DNS ($640^2 \times 320$)

ILES (64^3)



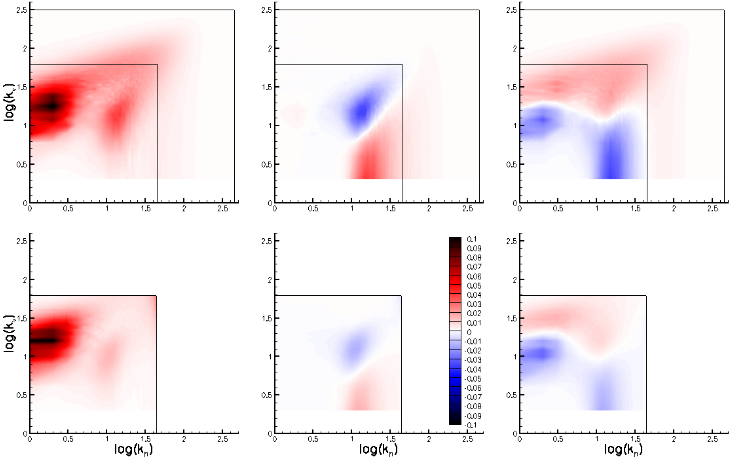
ILES: Turbulent Diffusion Spectra

Strongly stratified case ($\mathrm{Fr} = 0.025$, $\mathrm{Re}\,\mathrm{Fr}^2 = 3.74$)

$\mathcal{T}(E_h)$ $\mathcal{T}(E_v)$ $\mathcal{T}(E_p)$

DNS ($640^2 \times 320$)

ILES (64^3)



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- detailed analysis of homogeneous stratified turbulence
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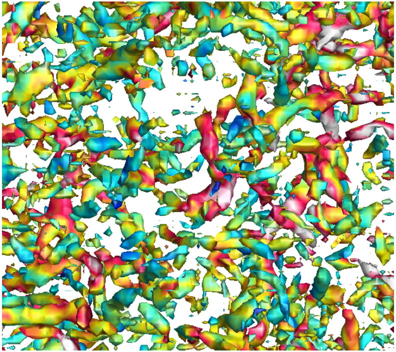
Summary and Outlook

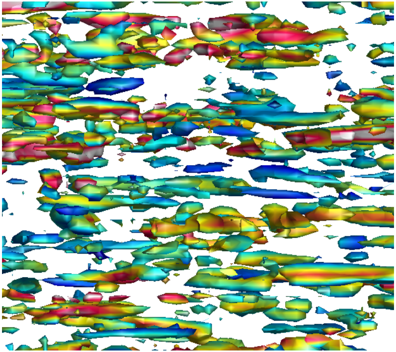
- detailed analysis of homogeneous stratified turbulence
- horizontal and potential energy concentrate near vertical axis
- vertical kinetic energy is more isotropically distributed
- implicit LES with ALDM predicts strongly stratified spectra much better than Smagorinsky model
- present results were obtained without recalibrating the model coefficients for stratified turbulence

Future work:

- analysis of ALDM in stratified media using the statistical methods of Horenko, Klein and Munz; possible re-calibration of the model constants
- application of ALDM to the simulation of breaking gravity waves

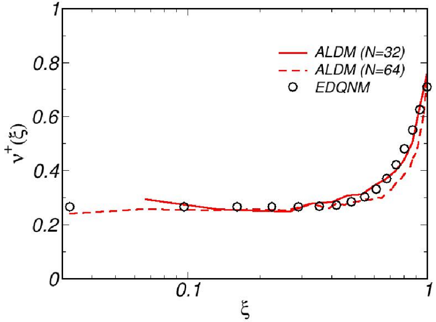
Thank you for your attention





ALDM: Model Calibration

- 5 independent model parameters
- optimization by evolutionary algorithm
- case: isotropic turbulence at $\mathrm{Re} = \infty$
- cost function: deviation from spectral eddy viscosity (EDQNM theory)

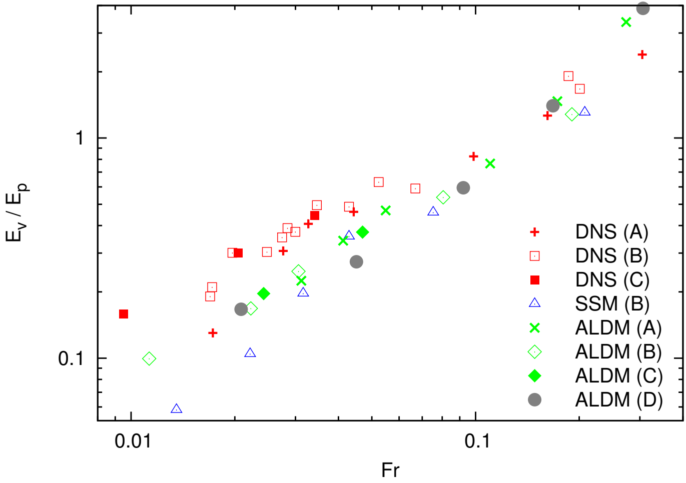


S. Hickel, N. A. Adams & J. A. Domaradzki: An Adaptive Local Deconvolution Method for Implicit LES, *Journal of Computational Physics*, 213, pp. 413-436, 2006

DNS: Energy Transport and Conversion

	E_h	E_v	E_p	scale range
Forcing $\mathcal{F}$	$\oplus$			large scales
Turbulent diffusion $\mathcal{T}$	inter-scale transfer			
Pressure transport $\mathcal{P}$	$\ominus$	$\oplus$		medium scales
Buoyancy transport $\mathcal{B}$		$\ominus$	$\oplus$	?
Molecular dissipation $\mathcal{D}$	$\ominus$	$\ominus$	$\ominus$	small scales

ILES: Potential to Vertical Energy Ratio



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D: $Re = \infty$