Large Eddy Simulation with Adaptive Moving Meshes

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Outline

Scientific Goals

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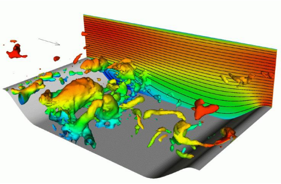
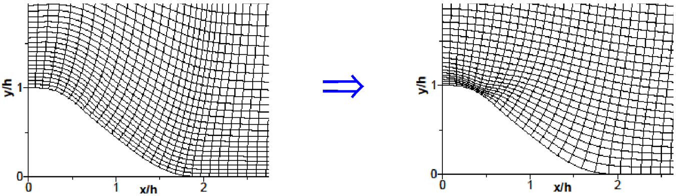
- Optimizing LES via r-adaption ("redistribution")
- Appropriate monitor functions for LES
- Multi-scale modelling with variable filter width
- Adaptive scale separation





Setup

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- physical domain Ω with coordinates $\mathbf{x} = (x_1, \dots, x_n)^T$
- computational domain Ω_c with coordinates $\boldsymbol{\xi} = (\xi_1, \dots, \xi_n)^T$
- solution of the physical PDE $\mathbf{u} = \mathbf{u}(\mathbf{x}, t)$
- 1-1 coordinate transformation $\mathbf{x} = \mathbf{x}(\boldsymbol{\xi}, t)$
- minimise mesh adaptation functional

$$\mathcal{I}[\xi] = \frac{1}{2} \int_\Omega \sqrt{g} \left(\sum_i \nabla \xi_i^T G^{-1} \nabla \xi_i \right) \mathrm{d}x$$

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$$\mathcal{I}[\xi] = \theta \int_\Omega \sqrt{g} \left(\sum_i \nabla \xi_i^T G^{-1} \nabla \xi_i \right)^{\frac{nq}{2}} \mathrm{d}x + (1 - 2\theta) n^{\frac{nq}{2}} \int_\Omega \frac{\sqrt{g}}{(\|J\| \sqrt{g})^q} \mathrm{d}x$$

The Mesh Moving PDE (MMPDE)

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Interchanging the roles of dependent and independent variables in the (modified) gradient flow equation of $\mathcal{I}[\xi]$ leads to the MMPDE:

$$\tau \frac{\partial \mathbf{x}}{\partial t} = P \left[\sum_{i,j} a_{ij} \frac{\partial^2 \mathbf{x}}{\partial \xi_i \partial \xi_j} - \sum_i b_i \frac{\partial \mathbf{x}}{\partial \xi_i} \right],$$

where

$$a_{ij} = (\nabla \xi_i) \cdot G^{-1} (\nabla \xi_j), \qquad b_i = \sum_j (\nabla \xi_i) \cdot \frac{\partial G^{-1}}{\partial \xi_j} (\nabla \xi_j).$$

$G(\Psi)$: Monitor function depending on some QoI Ψ

The Monitor Function $G(\Psi)$

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Monitor functions for multiple QoIs:

$$G(\Psi) = \omega(\Psi) I,$$

where

$$\omega(\Psi) = \sqrt{1 + \frac{\alpha}{\Psi_M} \sum_{k=1}^{N} \left(\frac{\Psi_k}{\Psi_{k,max}} \right)^2}$$

or

$$\omega(\Psi) = 1 + \frac{\alpha}{\sum_{k=1}^{N} w_k} \sum_{k=1}^{N} w_k \left[\frac{(1-\beta)\alpha_k + \beta \|\Psi_k\|}{(1-\beta)\alpha_k + \beta M_k} \right]$$

Moving Meshes in LESOCC2

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Finite Volume Method

Filtered ALE formulation for mass and momentum conservation

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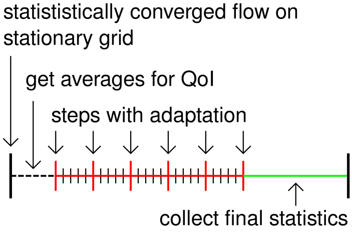

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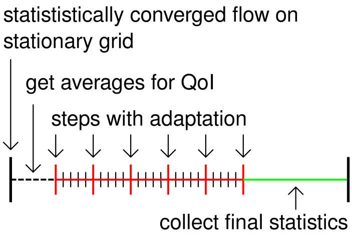

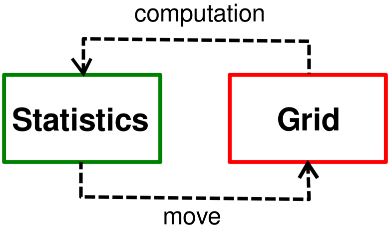

Physically Motivated QoIs

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$< . >:=$ average in homogeneous direction and time

1. Gradient of streamwise velocity: $\Psi = \nabla < u >$
2. Modelled TKE: $\Psi = \frac{< k_{sgs} >}{< k_{res} > + < k_{sgs} >}$
3. Turbulent viscosity: $\Psi = < \nu_t >$
4. Turbulent shear: $\frac{< \tau_{12}^{mod} >}{< \tau_{12}^{mod} > + < u' v' >}$

Results

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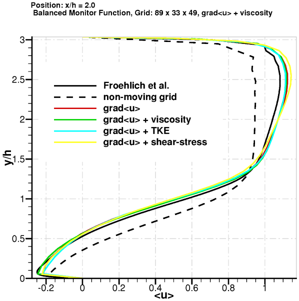
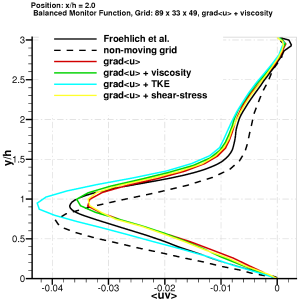
Flow over periodic hills ($Re = 10595$)

- Grid 1: $196 \times 128 \times 186 \rightarrow \approx 4.6$mio points (—)
- Grid 2: $89 \times 33 \times 49 \rightarrow \approx 144$k points (- -)





Production of TKE as QoI

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Idea: resolve production – model dissipation

Time-averaged production: $-\sum_{ij} < u_i', u_j' > \frac{\partial < u_i >}{\partial x_j}$

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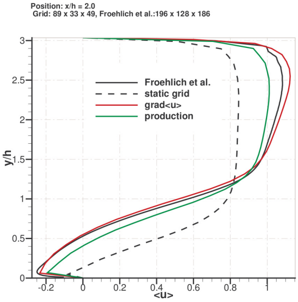

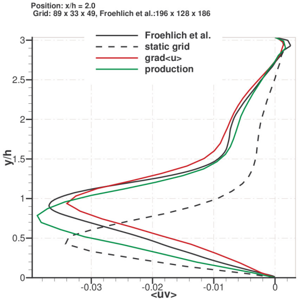

Summary and Current Research

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- High potential of mesh moving methods based on adaptive scale separation
- Resolve as much physics as possible with given DoFs
- Extension to coupled turbulence and wave propagation

  $\Rightarrow$ rotating annulus (Egbers/Harlander experiment)
- Develop efficient strategies for instationary scale separation
- Sensitivity-based dynamic mesh movement w.r.t. QoI