Stochastic Parametrisation in Weather and Climate Models:
Towards the prototype probabilistic earth-system model

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Towards Comprehensive Earth System Models

1970
Atmosphere
Land surface
Ocean & sea-ice

1985
Atmosphere
Land surface
Ocean & sea-ice

1992
Atmosphere
Land surface
Ocean & sea-ice
Sulphate aerosol

1997
Atmosphere
Land surface
Ocean & sea-ice
Sulphate aerosol
Non-sulphate aerosol
Carbon cycle

2000
Atmosphere
Land surface
Ocean & sea-ice
Sulphate aerosol
Non-sulphate aerosol
Carbon cycle
Atmospheric chemistry

Off-line model development
Strengthening colours denote improvements in models

The MetOffice Hadley Centre
“...models still show significant errors. Although these are generally greater at smaller scales, important large-scale problems also remain. ...The ultimate source of most such errors is that many important small-scale processes cannot be represented explicitly in models, and so must be included in approximate form as they interact with larger-scale features.

...consequently models continue to display a substantial range of global temperature change in response to specified greenhouse gas forcing. “
Traditional computational ansatz for Earth-System models

\[ \rho \left( \frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla \right) \mathbf{u} = \rho \mathbf{g} - \nabla p + \nu \nabla^2 \mathbf{u} \]

Increasing scale

Eg momentum “transport” by:
- Turbulent eddies in boundary layer
- Orographic gravity wave drag.
- Convective clouds

Deterministic local bulk-formula parametrisation

\[ P \left( X_n ; \alpha \right) \]
Deterministic bulk-formula parametrisation presumes a large ensemble of eg deep convective cloud systems within a grid box, in quasi-equilibrium with the large-scale flow.

Similar considerations for other parametrised processes, eg orographic gravity wave drag
Observations indicate a (shallow) power law for atmospheric energy wavenumber spectra, indicating no scale separation between resolved and unresolved scales in weather and climate models.
Let \( v, p \) be a solution to the Navier Stokes equations. Then, for any \( \tau \in \mathbb{R}^+ \),

\[
\begin{align*}
 v_\tau(x, t) &= \tau^{-1/2} v \left( \frac{x}{\tau^{1/2}}, \frac{t}{\tau} \right) \\
p_\tau(x, t) &= \tau^{-1} p \left( \frac{x}{\tau^{1/2}}, \frac{t}{\tau} \right)
\end{align*}
\]

is also a solution pair

...but violated by conventional deterministic parametrisations
Let $E(k)$ denote the kinetic energy per unit wave number of the system at wave number $k$.

Define an "eddy turn-over time"

$$\tau(k) \sim \frac{L}{U} \sim k^{-3/2} E^{-1/2}(k)$$
Suppose we are only interested in simulating some low wavenumber (ie large-scale) $k_L$.

How long before small-scale errors affect this large scale?

Let the time $\Omega$ taken for a small-scale error, confined to wavenumbers greater than $2^N k_L$, to grow and nonlinearly infect $k_L$ be given by

$$\Omega(N) = \tau(2^N k_L) + \tau(2^{N-1} k_L) + \ldots + \tau(2^0 k_L)$$

$$= \sum_{n=0}^{N} \tau(2^n k_L)$$
If $E(k) \sim k^{-5/3}$ then $\tau(k) \sim k^{-2/3}$ and

$$\Omega(N) = \sum_{n=0}^{N} \tau(2^n k_L)$$

$$= \tau(k_L) \sum_{n=0}^{N} 2^{-2n/3} \sim \tau(k_L) \text{ as } N \to \infty$$

Finite time for error in representation of small scales to affect accuracy of simulation of large scales, no matter how small in scale and hence amplitude this model error is (Lorenz 1969)
It is therefore not surprising that climate projections (even for large-scale variables) are uncertain.

How do we represent model uncertainty in climate projection? Are we confident we are representing uncertainty accurately?

There are currently two methods:

1. The multi-model ensemble (CMIP)
2. The perturbed parameter ensemble (e.g., climateprediction.net, UKCP09)
Shortcomings of the MME

• Insensitive to structural uncertainty (truncation/parametrisation ansatz)
• Limited ensemble sizes

Shortcomings of the PPE

• Very insensitive to structural uncertainty
• Large ensemble sizes, but how independent are the members of the ensemble, i.e., what is the effective ensemble size?
Given the importance of model error, it is unsatisfactory that representations of model error are treated in such an *ad hoc* fashion.

Another, potentially less *ad hoc* approach has been developed in NWP based on stochastic representations of sub-grid processes (Palmer, 1997, 2001; Buizza et al 1999).

Is it time to apply these ideas in climate prediction?
Traditional computational ansatz for Earth-System models

\[ \rho \left( \frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla \right) \mathbf{u} = \rho \mathbf{g} - \nabla p + \nu \nabla^2 \mathbf{u} \]

Deterministic formula to represent bulk effect of “ensemble” of sub-grid processes

Increasing scale
A stochastic-dynamic paradigm for climate models

Increasing scale

Coupled over a range of scales – to parametrise energy backscatter

Computationally-cheap nonlinear stochastic-dynamic model, providing specific realisations of sub-grid processes
Stochastically Perturbed Parametrisation Tendencies (Multiplicative Noise)

\[ X_p = (1 + r\mu)X_c \]

- Spectral pattern generator Spectral coefficients based on AR(1) processes.
- Clipped in boundary layer and stratosphere.
- Total deterministic parametrised tendency

\[ \sigma_1 = 0.5, \quad \tau_1 = 6\text{hrs}, \quad L_1 = 500\text{km} \]
\[ \sigma_2 = 0.2, \quad \tau_2 = 30\text{days}, \quad L_2 = 2500\text{km} \]
Realisations of $r$
Spectral Stochastic Backscatter Scheme

\[ F_\psi = \left( \frac{b_R D_{tot}}{B_{tot}} \right)^{1/2} \]

Streamfunction forcing

\[ F_\psi^* \]

Pattern using spectral AR(1) processes as SPPT

\( D_{tot} \) is a smoothed total dissipation rate, normalized here by \( B_{tot} \) and \( b_R \) is the backscatter ratio
Stochastic-Dynamic Cellular Automata

Eg for convection

EG Probability of an “on” cell proportional to CAPE and number of adjacent “on” cells – “on” cells feedback to the resolved flow

(Palmer; 1997, 2001; Shutts 2005; Berner et al, 2008)
Stochastic Parametrization and Model Uncertainty


Research Department

October 8, 2009
What are the benefits of a stochastic representation of unresolved scales?
1. More accurate probabilistic predictions
Experiments with the Lorenz ‘96 System

\[
\frac{dX_k}{dt} = -X_{k-1} (X_{k-2} - X_{k+1}) - X_k + F - \frac{hc}{b} \sum_{j=J(k-1)+1}^{kJ} Y_j
\]

\[
\frac{dY_j}{dt} = -cbY_{j+1} (Y_{j+2} - Y_{j-1}) - cY_j + \frac{hc}{b} X_{\text{int}[j-1/J+1]}
\]

Assume Y unresolved

Approximate sub-grid tendency by U

- Deterministic: \( U = U_{\text{det}} \)
- Additive: \( U = U_{\text{det}} + e_{w,r} \)
- Multiplicative: \( U = (1+e_r) U_{\text{det}} \)

Where:
- \( U_{\text{det}} = \) cubic polynomial in X
- \( e_{w,r} = \) white / red noise

Fit parameters from full model

Skill at lead time 5 “days”
Medium-Range Predictions of 850hPa Temperature
ENSEMBLES MME vs ECMWF stochastic physics ensemble (SPE)

lead time: 1 month

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Hindcast period: 1991-2005
SP version 1055m007

Weisheimer et al (2011)
2. Reduction in model bias
Stochastic parametrisation has potential to reduce climate model bias

Eg ball bearing in potential well.
\[
\frac{\partial \psi}{\partial t} = \Psi(x, y) \cdot \sqrt{rD}
\]

Berner et al. (2011)
3. More efficient use of limited human resources
It is often said, by way of justifying the continued institutional identity of climate models:

“We need to maintain a gene pool of diverse climate models in order to have credible estimates of uncertainty in predictions of climate change. “

But how big is this “gene pool”? How diverse are our climate models.
On the Effective Number of Climate Models

Pennell and Reichler. J.Clim. 2011

“For the full [CMIP] 24-member ensemble, this leads to an \( M_{\text{eff}} \) that...lies only between 7.5 and 9.”

“The strong similarities in model error structures found in our study indicate a considerable lack of model diversity. It is reasonable to suspect that such model similarities translate into a limited range of climate change projections.”
Perturbed parameter ensemble cluster together

Masson and Knutti, 2011

Surface Temperature
So, the gene pool is not that big.

Also, as well as being blind to structural errors in the standard deterministic truncation/parametrisation ansatz, maintenance of the current MME, whilst at the same time developing Earth-System complexity etc, places huge (impossible?) demands on human resources at the institutional level.
The notion of the Probabilistic Earth-System Model opens up the possibility of a more community-wide collaborative approach to model development?
Network on Stochastic Parameterization and Modelling

- Initiated at a recent Isaac Newton Institute programme on mathematics and climate
- Moderated by Judith Berner (NCAR) and Tim Palmer, (Univ. of Oxford, ECMWF)
- URL has info on how to subscribe and post messages and get help from the site administrator
- Every member can post to list
- Sign up at http://mailman.ucar.edu/mailman/listinfo/stoch
4. More efficient use of computing hardware
Don Grice. IBM Chief Engineer
“Application Scaling in an Exascale Environment”

“There will be a tension between energy efficiency and error detection.”

I.e., in future, if you insist on exact bit reproducibility, you will pay an enormous energy premium.

End of the deterministic bit-reproducible paradigm for HPC in sight?
MIT Spin-out Lyric Semiconductor Launches a New Kind of Computing with Probability Processing Circuits
Future Technology to Enable 1,000X Performance Over Today’s Digital Processors
Lyric Semiconductor, Inc. a DARPA- and venture-funded MIT spin-out, today launch a new technology called probability processing, which is poised to deliver a fundamental change in processing performance and power consumption. For over 60 years, computers have been based on digital computing principles. Data is represented as bits (1s and 0s). Lyric has invented a new kind of logic gate circuit that uses transistors as dimmer switches instead of as on/off switches. These circuits can accept inputs and calculate outputs that are between 0 and 1, directly representing probabilities - levels of certainty.
Although deterministic modelling of fluids has a long and venerable history, stochastic closures are more consistent with the work of:

“I believe that the ultimate climate models...will be stochastic, ie random numbers will appear somewhere in the time derivatives” Lorenz 1975.
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