Irregularity and singular vector growth of the differentially heated rotating annulus flow

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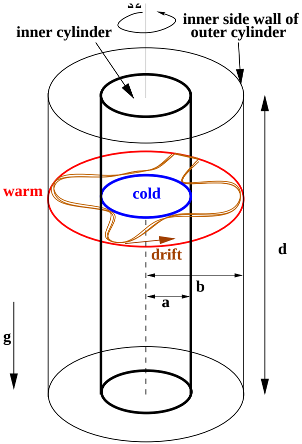
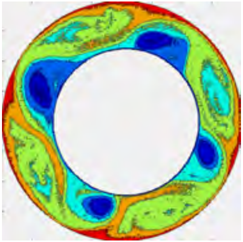
Content

Annulus experiment at BTU Cottbus



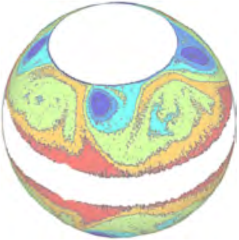
Sketched by Th.v.Larcher

surface temperature



cloud cover

Projection on sphere



surface temperature

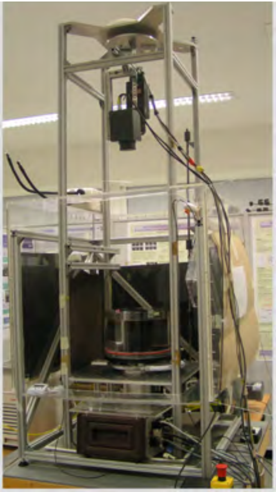
Equations, numbers, and experimental setup

$$\frac{d\mathbf{v}}{dt} = -\nabla p + \nabla^2 \mathbf{v} - \mathrm{Ra}\,\theta\mathbf{k} - \mathrm{Ta}^{1/2}\mathbf{k}\times\mathbf{v}$$

$$\frac{d\theta}{dt} = \frac{1}{\mathrm{Pr}}\nabla^2\theta$$

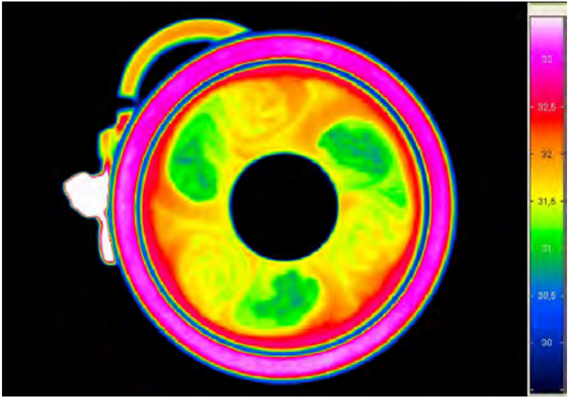
$$\nabla\cdot\mathbf{v} = 0$$

Taylor	Ta	$\frac{4\cdot\Omega^2\cdot(b-a)^4}{\nu^2}$
Rayleigh	Ra	$\frac{g\cdot\alpha\Delta T\cdot(b-a)^3}{(\kappa\cdot\nu)}$
Prandtl	Pr	$\frac{\nu}{\kappa}$
mod. Taylor	Ta’	$\frac{b-a}{d}$ Ta
th. Rossby	Ro	$4\frac{Ra}{PrTa}$



Animation of the wave spin-up

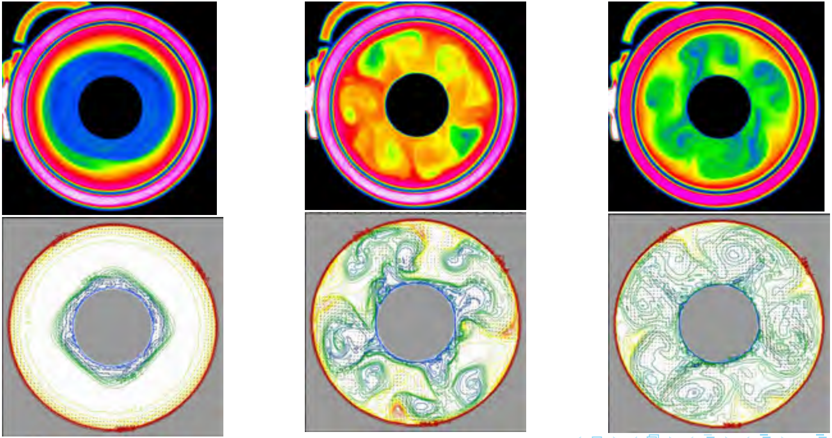
	Ω [rpm]	ΔT [K]	Ta	Ro_{th}	$b - a$ [mm]	d [mm]
exp	6	5.1	$1.55 \cdot 10^7$	0.79	75	135



Spin-up of a baroclinic wave: comparison EULAG and experiment

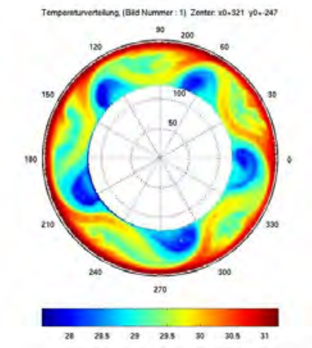
	Ω [rpm]	ΔT [K]	Ta	Ro_{th}	$b-a$ [mm]	d [mm]
exp	10	4.7	$3.77 \cdot 10^7$	0.28	75	135
num	48	6.7	$1.74 \cdot 10^9$	0.012	75	135

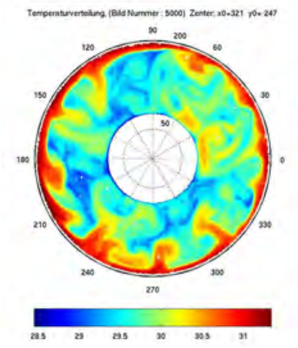
surface snapshots at 300s, 411s, 465s (exp) and 225s, 250s, 275s (num)



Data base

Shown is the surface temperature of a slow ($b - a = 50mm$, $Ro = 1.38$, $Ta = 3.62 \cdot 10^6$, left), and a fast rotating case ($b - a = 75mm$, $Ro = 0.029$, $Ta = 6.88 \cdot 10^8$, right). The transition between wave regimes is due to baroclinic instability. The gradual transition to turbulence might be related to non-classical local instability. Can we estimate the patterns of non-classical instability from the data?





How can we find the most 'unstable' initial condition?

In general a linear system can be written as

$$\frac{d}{dt}\mathbf{u} = \mathbf{A}\mathbf{u}. \tag{1}$$

The formal solution of the linear system (1) reads

$$\mathbf{u}(t) = \mathbf{P}\ \mathbf{u}(t_0), \qquad \mathbf{P} = e^{(t-t_0)\mathbf{A}}, \qquad t_0 = 0. \tag{2}$$

The matrix **P** is called the propagator of the linear system. The singular vectors are the eigenvectors of

$$\mathbf{M}\mathbf{v} = \lambda\mathbf{v}, \qquad \text{where} \qquad \mathbf{M} = \mathbf{D}^{-1}\,\mathbf{P}^{\mathbf{T}}\,\mathbf{D}\,\mathbf{P}, \tag{3}$$

D is the identity matrix and $\mathbf{P}^{\mathbf{T}}$ is the transpose of **P** (Borges and Hartman, 1992).

How can we estimate the system matrix from data?

Let us assume that we do not know (1) but instead that we have just data $\mathbf{u}(i)$. We propose that the data are consistent with the linear process

$$\mathbf{u}(i+1) = \mathcal{P}\ \mathbf{u}(i).$$

Then, by following Hasselmann (1988), the propagator is estimated as

$$\mathcal{P} = \Sigma_1 \Sigma_0^{-1} \approx \mathbf{P}, \tag{4}$$

where

$$\Sigma_j = \frac{1}{N} \sum_i^N (\mathbf{u}(i+j) - \bar{\mathbf{u}})(\mathbf{u}(i) - \bar{\mathbf{u}})^T. \tag{5}$$

Here $j = 0, 1$, and $\bar{\mathbf{u}}$ is the time mean.

Estimation using real data

Let us consider the following test:

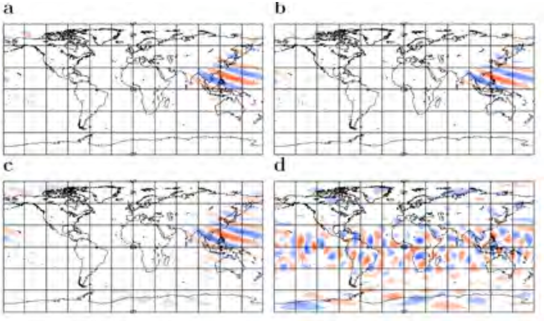
i We use the barotropic vorticity equation on the sphere.

ii The equation is linearized about an observed atmospheric mean state $\bar{\psi}$. The mean state is the winter mean (DJF) of the NCEP 500hPa data from 1958-2008.

iii We set $\tau_D = 5$ days, $K_H = 2.34 \cdot 10^{16} m^4 s^{-1}$, that are reasonable values for atmospheric flows.

Quality of system matrix estimation

We compare the known with the estimated singular vectors.

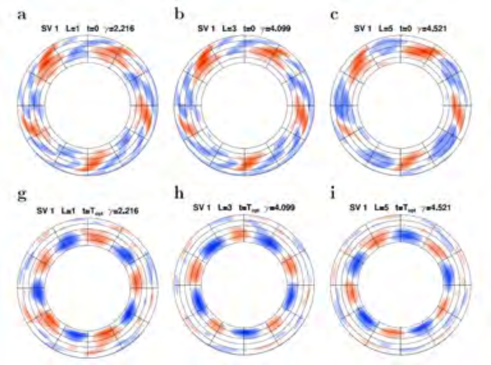


Shown are the leading SVs for $t_{opt} = 48h$. a) reference SV, b) experiments with 1000, c) 750, and d) 500 winters.

Surrogate data: $\psi_{i+1} = \mathbf{P}\psi_i + \mathbf{f}_i$

Data from the narrow gap experiment

Comparison of the fastest growing patterns of the narrow gap experiment for different optimization times (1,3,5 full rotations). Noise is reduced by EOF filtering (80% of the variance have been kept). $t = 0$ in a-c, $t = t_{opt}$ in g-i.

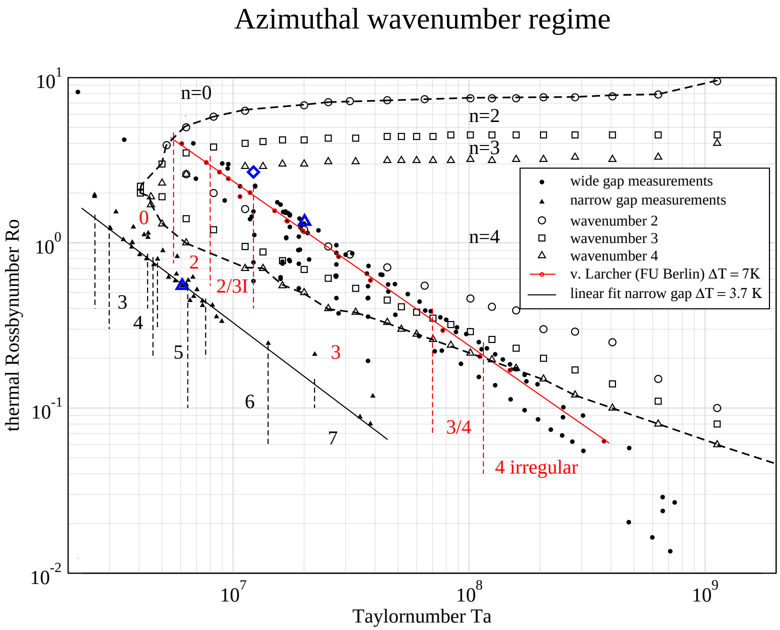


For the irregular flow we so far did not get useful results. The main reason is probably the insufficient length of the data sets.

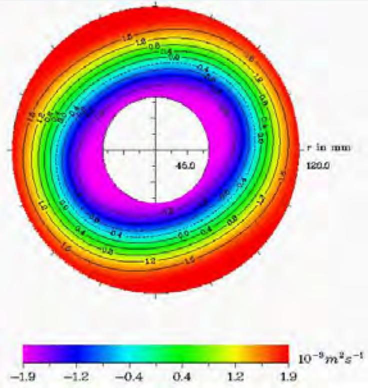
Spectral Lorenz-Model

- Modification of Phillips 2-Layer-Model
- Coriolis parameter $f = 2\Omega = const.$
- Parameter for stability $\sigma = \sigma(t)$

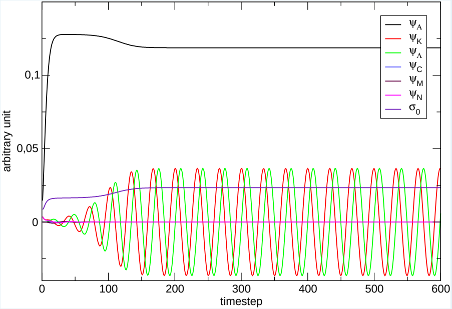
0 hPa	———————	
250 hPa	- - - - - - - - - - - -	$\psi + \tau,\ \theta + \sigma,\ -\chi$
500 hPa	———————	$\psi,\ \theta$
750 hPa	- - - - - - - - - - - -	$\psi - \tau,\ \theta - \sigma,\ \chi$
1000 hPa	———————	


Azimuthal wavenumber regime
thermal Rossbynumber Ro
Taylornumber Ta
10^1
10^0
10^{-1}
10^{-2}
10^7
10^8
10^9
n=0
n=2
n=3
n=4
0
2
2/3I
3
3/4
4 irregular
3
4
5
6
7
wide gap measurements
narrow gap measurements
wavenumber 2
wavenumber 3
wavenumber 4
v. Larcher (FU Berlin) ΔT = 7K
linear fit narrow gap ΔT = 3.7 K

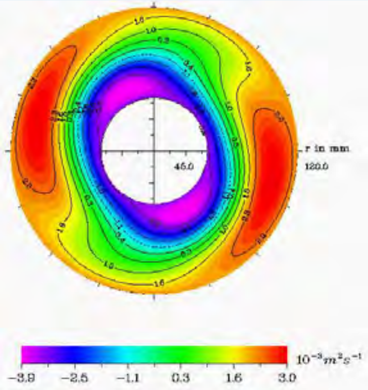
1st mode



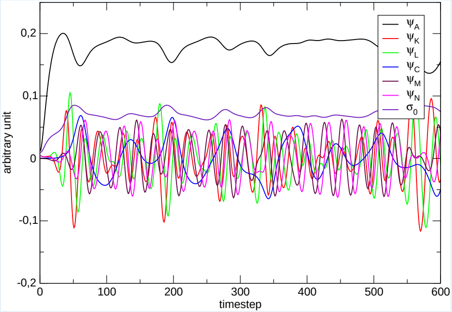
spectral coefficients



irregular mode



spectral coefficients

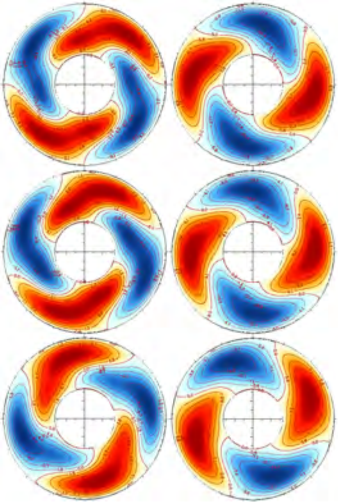


Examples of singular vectors

Wavenumber 2 singular vector (left) and wavenumber 5 singular vector (right) at the upper layer, the interface and the bottom layer.

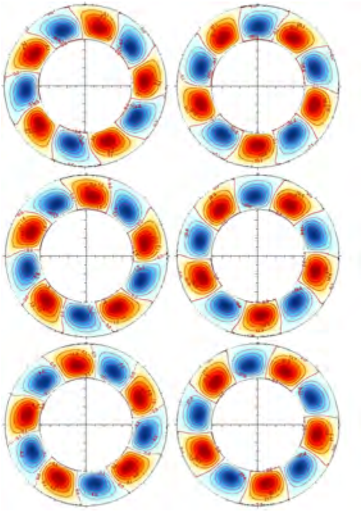
$Ta = 1.3 \cdot 10^7$, $Ro = 2.8$

$t = 0$ $\qquad$ $t = t_{opt}$

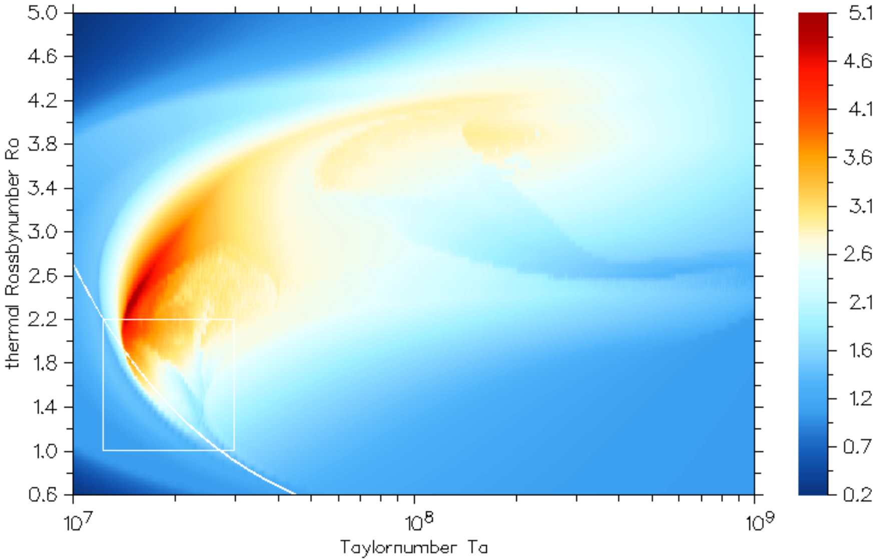


$Ta = 2.0 \cdot 10^7$, $Ro = 1.35$

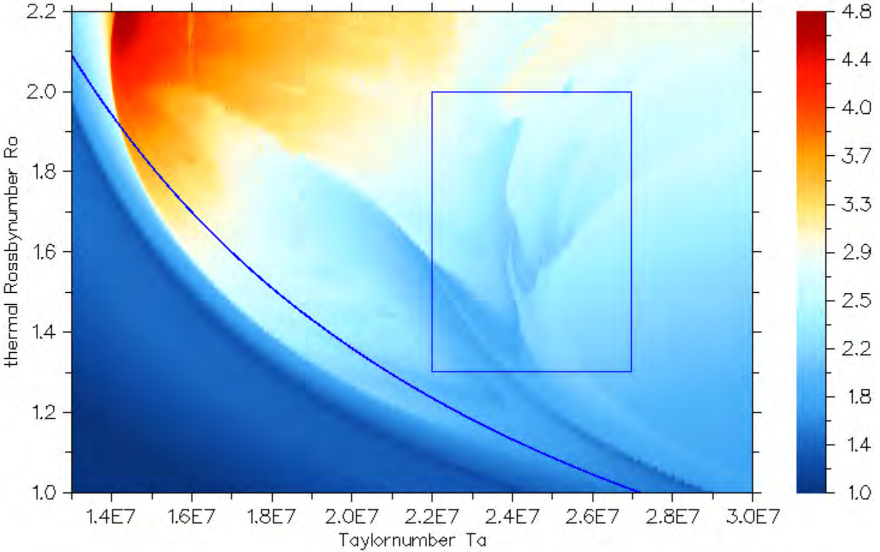
$t = 0$ $\qquad$ $t = t_{opt}$

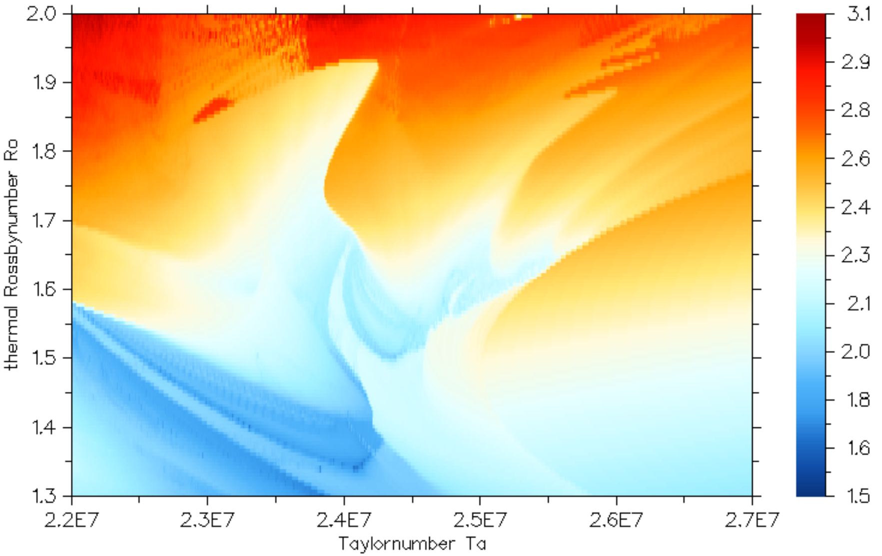


Regime diagram in terms of 1st singular value

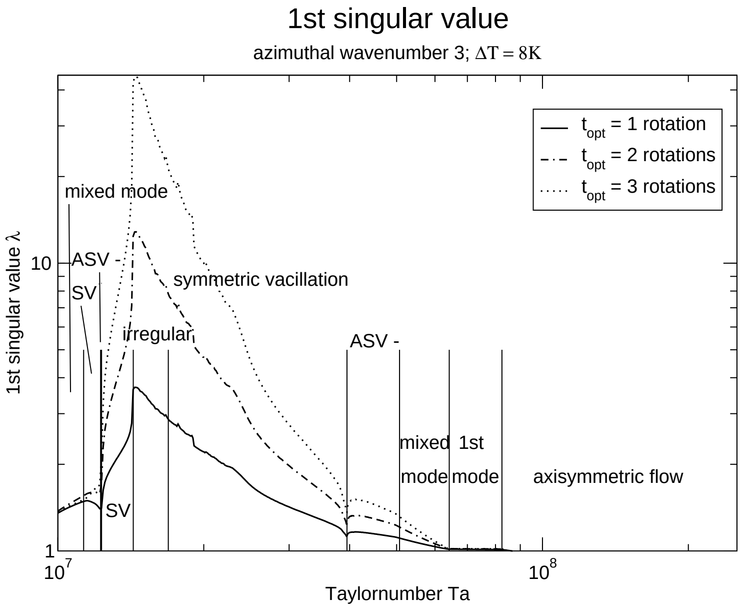


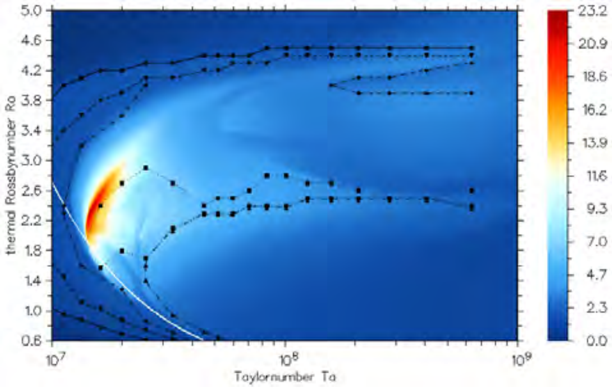
Regimes and 1st singular value





2.0
1.9
1.8
1.7
1.6
1.5
1.4
1.3
thermal Rossbynumber Ro
2.2E7
2.3E7
2.4E7
2.5E7
2.6E7
2.7E7
Taylornumber Ta
3.1
2.9
2.8
2.6
2.4
2.3
2.1
2.0
1.8
1.6
1.5


1st singular value
azimuthal wavenumber 3; ΔT = 8K
t_{opt} = 1 rotation
t_{opt} = 2 rotations
t_{opt} = 3 rotations
mixed mode
ASV -
SV
symmetric vacillation
irregular
SV
ASV -
mixed mode
1st mode
axisymmetric flow
1st singular value λ
10
1
10^7
10^8
Taylornumber Ta



Solid line - transition Hadley mode to Rossby 1st mode; dashed line - transition Rossby 1st mode to Rossby mixed mode; dashed dotted line - transition Rossby mixed mode to SV, ASV(+,-) resp.; dotted line - transition to irregular ow; dashed double dotted line - transition to Rossby 2nd mode.

Results and future work

We used simple models and laboratory data to study singular vector growth in the rotating differentially heated annulus.

Main results:

i Simple test cases show that SVs can be estimated from data

ii The most serious problem is that data sets have to be long

iii The method might thus be less useful for field data but is promising for long laboratory data sets

iv The differentially heated rotating annulus is an interesting case since non-classical instability might control the transition to irregular flows

v Irregular regimes have by far the largest growth rates

vi It appears that SV growth factors can be used to quantify irregularity

Thank you for attention!

Thanks for support

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